PRIMARY ALGEBRA

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MacDonald's Mathematical Series.

PRIMARY ALGEBRA.

STUDENT'S MANUAL.

BY

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Boston:
ALLYN AND BACON.
1896.

MACDONALD'S PRIMARY ALGEBRA.

TYPOGRAPHY AND ELECTROTYPING BY C. J. PETERS & SON,
145 HIGH ST., BOSTON, U.S.A.

CONTENTS.

PRIMARY ALGEBRA.

To the scholars. — Strive to work out the problems in these lessons without help.

LESSON I.

Problems to be Solved by the Use of x.

1. Four times, seven times, and three times a number is 294; what is the number?

2. Three men, A, B, and C, divide $288 in such a way that B has twice as much, and C five times as much as A; how much has each?

3. A post 28 feet long stands six times as much above ground as in the ground; how deep is it in the ground? How long above ground?

4. A general distributed 2400 men in three forts so that fort No. 2 had twice as many as fort No. 1, and fort No. 3 had as many as in both the others; how many did he put in each fort?

5. Divide the number 534 into two parts, so that one part will be five times the other.

6. A string 125 inches long is cut so that one piece is four times as long as the other; how long is each?

7. A man bought a horse, a carriage, and a harness for $225. He gave four times as much for the carriage

as for the harness, and as much for the horse as he did for both the carriage and the harness; how much did he give for each?

8. $3x + 4x + 6x = 403$, to find value of x.

9. $5x + 2x = 154$, to find value of x.

10. I bought a number of apples for two cents each, and the same number of oranges for three cents each, paying 55 cents for the whole; how many did I buy?

LESSON II.

Addition and Subtraction of Positive Quantities — Problems.

1. Three men, A, B, and C, had 27 m acres of land, of which B had three times as much as as A, and C five times as much as A; how many had each?

2. A boy had some oranges, three times as many apples, and as many pears as apples and oranges together; the whole number was 240. How many had he of each?

3. Find the sum of the following quantities, and also the difference by subtracting the second from the first, arranging them thus:—

NO.	QUANTITIES.		SUM.	DIFFERENCE.
1	$4\,m$	$2\,m$	$6\,m$	$2\,m$
2	$25\,c$	$18\,c$	$43\,c$	$7\,c$

1. $7a$ and $4a$.
2. $9b$ and $3b$.
3. $12m$ and $8m$.
4. $8a$ and $3m$.
5. $6c$ and $5b$.
6. $25b$ and $22a$.
7. $4c + 8$ and $3c + 6$.
8. $9d + 12$ and $6d + 12$.
9. $5a + 7$ and $5a + 4$.
10. $5a + 6c$ and $3a + 3c$.
11. $7d + 8c$ and $4d + 8c$.
12. $12a + 9m$ and $12a + 8m$.

Sum and difference of

4. $13b + 7c + 18$ and $13b + 6c + 15$.
5. $5m + 6n + 7s - 8$ and $4m + 6n + 7s$.
6. $25a + 13b + 8c$ and $25a + 13b + 8c$.

7. Find the value of x in each of the following equations, if $a = 5$, $b = 4$, and $c = 10$.

1. $x = 6a$.
2. $x = 3a - 2b$.
3. $x = 4b - c$.
4. $x = 5a - 4b + c$.
5. $x = 3c - 5a - 4b$.
6. $x = 10a - 4b - 20$.
7. $x = 50 - 3c - 2b$.
8. $x = 10b - 4a - 20$.

8. Two railroad trains 165 miles apart are approaching each other, one at the rate of 30 miles, and the other 25 miles per hour; how long before they will meet?

9. Two ships are sailing towards each other at the rates of 8 and 12 miles per hour respectively. If the distance between them is 130 miles, how long before they will meet? How far will each have sailed?

10. In an orchard of 162 trees there are five times as many apple-trees as peach-trees, and half as many pear-trees as of both the other kinds together; how many are there of each?

11. To $5a + 6b + 2c$ add $a + 3b + 3c$; then subtract $4a + 8b + c$; then add $7a + 6b + 3c$; then subtract $9a + 7b + 6c$. What is the remainder?

12. Take $4a$; add $3a + 7b$; subtract $5b$; add $3a + 8b + 2c$; add $2a + 8b + 5c$; subtract $9b + 6c$; subtract $6a + 5b$; add $2a + 4b + 7c$; divide by 8. *Ans.?*

13. Form the following quantities:

1. One 6 larger than x; one 6 times x.
2. One 8 larger than a; one 8 times a.
3. One 5 smaller than x; one 5 times x.
4. One 7 smaller than b; one 7 times b.
5. One 10 larger than x; one 10 times x.

14. Form quantities:

1. a larger than x; a smaller than x.
2. b larger than m; b smaller than m.
3. $3b$ larger than y; $3b$ smaller than y.
4. $3c$ larger than $2b$; 3c smaller than $2b$.

15. Form numbers:

1. $a + b$ larger than $b + c$.
2. $m + n$ larger than $3m$.
3. $a + c$ smaller than $2a + c$.
4. $2a + 3b + 6$ smaller than $2a + 4b + 12$.

LESSON III.

Multiplication and Division — Exponents.

1. Form quantities:

1. a larger than y; a smaller than y.
2. $2b$ larger than $3a$; $2b$ smaller than $3a$.
3. $4m$ larger than $4x$; $4m$ smaller than $4x$.

2. Answers to the following:

1. a^4 multiplied by a^3; a^4 divided by a^3.

2.	a^5	multiplied by a^2;	a^5	divided by a^2.
3.	b^5	multiplied by b^3;	b^5	divided by b^3.
4.	$7\,c^2$	multiplied by $5\,c$;	$75\,c^2$	divided by $5\,c$.
5.	$125\,s^4$	multiplied by $25\,s^3$;	$125\,s^4$	divided by $25\,s^3$.
6.	$6\,m^2$	multiplied by $3\,m^2$;	$6\,m^2$	divided by $3\,m^2$.
7.	$12\,b^5$	multiplied by $6\,b^4$;	$12\,b^5$	divided by $6\,b^4$.

3. Five times a certain number and eight times the same number is 156; what is the number?

4. John has a number of cents, Henry has three times as many, Willie has as many as John and Henry together, and Fred has three-fourths as many as the other three; altogether they have $1.68. How many has each?

5. Add $5\,a^2$, $7\,a^2$, $3\,a^2$, $4\,a^2$, and $9\,a^2$ together, and multiply the sum by $3\,a^3$.

6. Add $6\,x^3$, $9\,x^3$, $2\,x^3$, x^3, $3\,x^3$, and $7\,x^3$ together, and divide the sum by $14\,x^2$.

7. Add together $5\,b$, $7\,b$, $8\,b$, $9\,b$, $6\,b$, and $5\,b$, and divide the sum by $20\,b^2$.

8. Add together $4\,c$, $8\,c$, $5\,c$, $6\,c$, and $2\,c$, and multiply the sum by $5\,c^3$.

9. Lucy has 15 cents more than Mary, and both together have 47 cents; how many have each?

10. In an election one candidate received 79 votes more than the other out of a total vote of 1,637; how many did each receive?

11. The difference between two numbers is 23, and their sum is 71; what are the numbers?

12. In an election A received a certain number of votes, B received 25 more than twice as many, and C 8 less than three times as many. The total vote was 3,227; how many did each receive?

LESSON IV.

Multiplication and Division continued — Problems.

1. A man's age is five times his son's age, the sum of their ages is 36; what is the age of each?

2. If three times and five times a number be added to the number, the sum will be $279\ m$; what is the number?

3. Form the following quantities:

1.	4 larger than a;	4 times a.
2.	a larger than x;	a times x.
3.	ab larger than y;	ab times y.
4.	d smaller than x;	d times x.
5.	a^2d smaller than a^2;	a^2d times a^2.

4. Do the following examples:

1.	a multiplied by b;	a divided by b.
2.	a^2x multiplied by a^2b;	a^2x divided by a^2b.
3.	b^2 multiplied by b^2;	b^2 divided by b^2.
4.	a^2bx multiplied by abx;	a^2bx divided by abx.
5.	a^4x^2 multiplied by a^5x;	a^4x^2 divided by a^5x.
6.	m^2 multiplied by m^3;	m^2 divided by m^3.
7.	$4\,x^3$ multiplied by $2\,x^2$;	$4\,x^3$ divided by $2\,x^2$.
8.	$16\,a^2x$ multiplied by $8\,a^3$;	$16\,a^2x$ divided by $8\,a^3$.
9.	$25\,b^3$ multiplied by $5\,b^3d^2$;	$25\,b^3$ divided by $5\,b^3d^2$.

5. Multiply $a^2x + ax + x$ by a^3x^2.

6. Divide $15\,a^3b + 25\,a^2b^2 + 15\,ab^3$ by ab.

7. Divide $5\,a + 10\,b$ by ab.

8. Multiply $3\,ab + 4\,a^2b^2 + 5\,a^3b^3$ by $2\,a^2b^2c$.

9. Multiply $9\,ax + 7\,a^2x + 6\,a^3x^2$ by $4\,ax^2$.

10. Divide $4\,m^2n + 6\,mn^2 + 8\,mn$ by $2\,mn$.

11. Multiply $3\,a + 4\,b + 5\,c$ by abc.

12. $3\,x + 25 = 109$, to find the value of x.

13. $5\,x - 60 = 80$, to find the value of x.

14. $6\,x + 12 = 78$, to find the value of x.

LESSON V.

Review — Negative Quantities.

Add the following:

1.	2.	3.	4.
$3\,x$	$5\,a^2$	$7\,bc^2$	$12\,x^2y$
$7\,x$	$6\,a^2$	$9\,bc^2$	$15\,x^2y$
$4\,x$	$7\,a^2$	$18\,bc^2$	$18\,x^2y$
$5\,x$	$8\,a^2$	$22\,bc^2$	$25\,x^2y$
$6\,x$	$4\,a^2$	$36\,bc^2$	$9\,x^2y$
$7\,x$	$3\,a^2$	$8\,bc^2$	$12\,x^2y$
$2\,x$	$9\,a^2$	$4\,bc^2$	$42\,x^2y$

5. From $9\,x^2y + 8\,ab + 19\,c^2$ subtract $8\,x^2y + 8\,ab + 12\,c^2$.

6. Add:

1. x to a^2.
2. b to c.
3. m to n.
4. $3\,b$ to $7\,a$.
5. $3\,a$ to $4\,b$.
6. $2\,c$ to d.

7. Subtract:

1. 9 from x.
2. $3\,b$ from a.
3. $2\,c$ from m.
4. $5\,d$ from 5.
5. $3\,x$ from $2\,d$.
6. 7 from $4\,x$.

8. Maine and Vermont together have an area of 42,605 square miles, and Maine has 23,475 square miles more than Vermont; what is the area of each?

9. Massachusetts and Connecticut have together an area of 13,305 sq. miles, and Massachusetts has 3,325 sq. miles more than Connecticut; what is the area of each?

10. New York, New Jersey, and Pennsylvania have together an area of 102,200 sq. miles. New York has 3,955 sq. miles more than Pennsylvania, and New Jersey has 37,400 sq. miles less; what is the area of each?

11. $x + 2a = 5a$, to find value of x in terms of a.

12. $3x + 6a = 9b$, to find value of x in terms of a and b.

13. $3x + 4a = 8a + x$, to find value of x in terms of a.

14. $5x - 7a = 2a + 9b + 2x$, to find value of x.

15. From the sum of $2m + 3n + 2x$, $7m + 3n + 5x$, $8m + 4x$, $3n + x$, and $5m + 2x$, subtract the sum of $3m + 8x$, $6n + 2x$, and $5m + 3n$.

16. Farmer A has five times as many bushels of wheat as farmer B, and farmer C has 10 bushels more than both of them. Altogether they have 82 bushels; how many has each?

17. Read these quantities:

$-7,\ -6,\ -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,$
$-7a,\ -6a,\ -5a,\ -4a,\ -3a,\ -2a,\ -a,\ 0,\ a,\ 2a,\ 3a,\ 4a,\ 5a,\ 6a,$
$-7b,\ -6b,\ -5b,\ -4b,\ -3b,\ -2b,\ -b,\ 0,\ b,\ 2b,\ 3b,\ 4b,\ 5b,\ 6b,$
$-7x,\ -6x,\ -5x,\ -4x,\ -3x,\ -2x,\ -x,\ 0,\ x,\ 2x,\ 3x,\ 4x,\ 5x,\ 6x.$

18. In the following pairs of quantities state which is the larger and how much; as, for example:

$5a$ is $7a$ larger than $-2a$.

6 or 2.	$5b$ or $7b$.	$7a$ or $3a$.
6 or -2.	$5b$ or $-7b$.	$-7a$ or $-3a$.

-6 or -2.	$9x$ or $-x$.	$-2a$ or $-9a$.
-5 or -2.	$-9x$ or x.	$2a$ or $9a$.
8 or 0.	$-12x$ or $-3x$.	$7a$ or $4a$.
-8 or 0.	$-2x$ or $7x$.	$-7a$ or $-4a$.
0 or $-6a$.	$9x$ or $8x$.	6 or -3.
$3b$ or $-3b$.	$-9x$ or $-8x$.	-6 or -3.

19. In the following give the smaller in the same way:

$7a$ or 0.	$9a$ or $-9a$.	$6b$ or $-7b$.
$-7a$ or 0.	0 or $-4x$.	$-8x$ or $-9x$.
-8 or 0.	0 or $-2b$.	$-8y$ or $8y$.

20. Write a quantity:[1]

6 larger than -6.	$2a$ larger than $-2a$.
2 larger than -7.	$6b$ larger than $-12a$.
7 larger than -7.	$9b$ larger than 0.
3 larger than -2.	$5x$ larger than $-3x$.
4 larger than -2.	$8x$ larger than $-12x$.
3 larger than -6.	$6b$ smaller than $-8b$.
$4b$ larger than $-4b$.	$7a$ smaller than 0.
$3a$ larger than $-7a$.	$7b$ smaller than $3b$.

Add:

21.	**22.**	**23.**	**24.**
$5a$	$5x$	$8b$	$12c$
$-2a$	$-3x$	$2b$	$-12c$
$-3a$	$-5x$	$-9b$	$-7c$
$-4a$	$8x$	$-3b$	$3c$
$6a$	$-4x$	$-7b$	$4c$
$-2a$	$3x$	$-2b$	$8c$
$3a$	$-4x$	$4b$	$-7c$

[1] Or give them orally.

LESSON VI.

Addition (Negative Quantities) — Problems.

Add the following:

1.	2.	3.	4.
9	$4a$	$2x$	$-12b$
-7	$-5a$	$-3x$	$210b$
2	$7a$	$-4x$	$375b$
8	$-2a$	$5x$	$-621b$
-4	$3a$	$6x$	$35b$
-5	$-6a$	$-7x$	$81b$
2	$-a$	$-5x$	$-273b$
7	$7a$	$-6x$	$836b$
6	$8a$	$3x$	$-239b$
3	$-2a$	$7x$	$47b$
-2	$3a$	$-2x$	$-99b$

NOTE. Do not add the positive and negative quantities separately, but in order as they occur. This is a good drill to cultivate watchfulness as to signs.

5.	6.	7.
$5ab$	$-9x^2$	$91ax^2$
$-7ab$	$12x^2$	$42ax^2$
$-31ab$	$-13x^2$	$-36ax^2$
$42ab$	$24x^2$	$-2ax^2$
$54ab$	$-54x^2$	$16ax^2$
$22ab$	$39x^2$	$-5ax^2$

8.	9.	10.
$5a^2$	$14ax$	$6y$
$-17a^2$	$-36ax$	$7y$
$-29a^2$	$97ax$	$8y$
$37a^2$	$-69ax$	$4y$
$-4a^2$	$-24ax$	$3y$
$-9a^2$	$32ax$	$8y$
$3a^2$	$91ax$	$9y$
$-3a^2$	$-27ax$	$7y$

11. $\frac{x}{2} + 9 = 25$, to find value of x.

12. $\frac{2x}{3} + \frac{1}{3} = 27$, to find value of x.

13. A farmer, when asked how many cows he had, replied that if he had as many more, and half as many more, he would have 75; how many had he?

14. A boy told his companions that if he had two-thirds as many cents more than he then had, plus 4 cents, he would have 49 cents; how many had he?

15. In a certain school of 84 scholars there are three-fourths as many boys as girls; how many are there of each?

16. A farmer has a number of cows in one herd, three-fourths as many in a second, and 4 more than half as many in a third; in all there are 76 cows. How many in each herd?

17. If to a certain number, itself and one-fifth of itself and 3 be added, it will give 80; what is the number?

18. Meeting a man driving a flock of geese to market, I said, "Good-morrow, master, with your hundred geese." Said he, "I have not a hundred; but if I had as many more, half as many more, and $2\frac{1}{2}$ geese, I would have a hundred." How many had he?

19. One-third of a number plus one-fourth of it is 7; what is the number?

NOTE. It may take two lessons to do all these examples, in which case spend a part of each lesson on drill in addition.

LESSON VII.

Addition continued — Problems.

Add:

1.

$$\begin{array}{r} 3x \\ -5x \\ 7x \\ -6x \\ 4x \\ -3x \\ \hline 5x \end{array}$$

2.

$$\begin{array}{r} 6a \\ -2a \\ -a \\ -7a \\ -5a \\ -3a \\ \hline 19a \end{array}$$

3.

$$\begin{array}{r} 6 \\ -7 \\ -3 \\ -2 \\ -4 \\ 8 \\ \hline -3 \end{array}$$

4.

$$\begin{array}{r} 2x+4 \\ 3x-2 \\ -4x+3 \\ -7x-2 \\ 8x-4 \\ 7x-6 \\ \hline \end{array}$$

5.

$$\begin{array}{r} 7x^2y-3ab^2+8d \\ -5x^2y-2ab^2-5d \\ 3x^2y+4ab^2+2d \\ -2x^2y+ab^2-d \\ 3x^2y-2ab^2+3d \\ -8x^2y+5ab^2-2d \\ \hline 3x^2y-6ab^2-5d \end{array}$$

6.

$$\begin{array}{r} 4x+c \\ -3x-c \\ 6x+4c \\ 5x-c \\ -7x-6c \\ -4x+2c \\ \hline -x+c \end{array}$$

Arrange and add the following:

7. $7x, 3a, -4b, -3x, 5a, -7b, -9c, 2x, -2a, 9c, 4b, -6x, 3a, 7b.$

8. $5a, -7, -2a, 3b, 5, 6a, -2.$

9. $3b-2a+6c,\ 2c-5b+2a,\ 3a-4c+3b, 7b-a+3c.$

10. $4a, -3c, 2b, -2a, 5c, -3b, -a, 2c.$

11. $8x, 3y, -4x, 3z, -4z, -7x, 3x, -3y, z.$

Find quantities:

12. a larger than x.
b larger than $x-b$.
c smaller than $x+c$.

$x + a$ larger than $x - a$.
$a + 3$ larger than $a - 3$.
$b - 3$ smaller than $b - 3$.

13. In a library two-fifths of the books are histories, one-third scientific works, and the remaining 240 are poems; how many volumes in the library, and how many of each kind?

14. Three-fourths of a river is in one State, one-eighth in another, and 24 miles of it in another; how long is the river?

15. $\frac{4x}{3} + \frac{3x}{4} + 50 = 568 - x$.

16. $x - 40 = 40 - \frac{x}{3}$.

17. A number is as much larger than 25 as 25 is larger than two-thirds of the number; what is the number?

18. A farmer sold two-fifths of his farm to one man, one-third to another, and had forty acres left; how large was his farm at first?

19. Henry had 8 marbles more than Fred, and Willie had 5 more than both the others; altogether they had 61 marbles. How many had each?

20. Add $7xy$, $-4ay$, $-3xy$, $-3ax$, $4ay$, $-4xy$, and ax.

LESSON VIII.

Subtraction — Miscellaneous Problems.

1. From 9 take 7.
2. From 7 take 9.
3. From -9 take -7.
4. From -7 take -9.

5. From $9a$ take $5a$.

6. From $5a$ take $9a$.

7. From $-9a$ take $-5a$.

8. From $-5a$ take $-9a$.

9.[1] From $7a^2 - 6ab + 4ac + 7c^2$, take $5a^2 - 9ab + 6ac + 7c^2$.

10. From $13m^2 + 8mn + 24n^2 - 9$ take $6m^2 + 8mn - 24n^2 - 18$, and add to the remainder $-6m^2 - 48n^2 - 9$.

NOTE. Rule a table of six columns as follows: —

No.	Quantities.		Sum.	1st — 2d.	2d — 1st.
	1st.	2d.			
1	$6a$	$4a$	$10a$	$2a$	$-2a$
2	$-8x^2$	$-5x^2$	$-13x^2$	$-3x^2$	$3x^2$

11. Fill out as above from the following quantities: —

1.	12	and 6.	11.	$24a^3$	and 7.
2.	-12	and -6.	12.	$36a^2$	and $12a$.
3.	-12	and 6.	13.	$5xy$	and $4c$.
4.	12	and -6.	14.	$8b$	and $8c$.
5.	$8a^2$	and $5a^2$.	15.	$7ax$	and $7ax$.
6.	$-8a^2$	and $-5a^2$.	16.	$9by$	and $-9by$.
7.	$13x^2y$	and $-7x^2y$.	17.	$3a^3$	and $4a^2$.
8.	$7ab$	and $-7ab$.	18.	$8x^2$	and $-8x^2$.
9.	m	and $-m$.	19.	12	and 12.
10.	$-7d$	and $-12d$.	20.	-12	and -12.

[1] In subtraction *do not change*, or *suppose changed*, the sign of the subtrahend, and then add. Subtract by finding the difference.

12. Rule a similar table, and fill out from the following binomial quantities: —

1. $a + b$ and $a - b$.
2. $x + 7$ and $x - 7$.
3. $6b + 3c$ and $7b - 9c$.
4. $5ab + 4x$ and $5ab - 4x$.
5. $y + x$ and $y - x$.
6. $3d + 9$ and $4d - 6$.
7. $8 + 2x$ and $8 - 2x$.
8. $12 + 3a$ and $7 - 4a$.
9. $13 + x$ and $13 - x$.
10. $15 - 9y$ and $14 - 9y$.

13. $x + 30 = \frac{x}{2} + 22$, to find value of x.

14. One-third of a man's property minus one-half of it equals 100 dollars; what is his property? How do you explain the answer? *Ans.* $\$ - 600$.

15. One day I deposited in a bank a certain sum of money, the next day I deposited the same sum plus 9 dollars; if I had deposited 43 dollars more, I would have deposited in all 30 dollars. How large was my deposit each day? Interpret the answer. *Ans.* $\$ - 11$.

16. A certain number and two-thirds of the number equals 45; what is the number?

17. Two-fifths of a road is through forests, one-third through fields, and $2\frac{2}{3}$ miles along a river bank; how long is the road?

18. From 0 subtract each of the following quantities:

$6m$; $-6m$; $-3ax$; $16a^2b$; $5ay$; $-8d$; $a - b$; $1 - a$.

19. Write a quantity $5a$ larger than $-10a$; $7x$ larger than $-4x$; 16 larger than -16.

20. What must be added to a to make it b? to $-a$ to make it b?

LESSON IX.*

Miscellaneous Problems.

Add:

1.	2.	3.	4.
$7x$	$75ab$	$345a^2$	$8b$
$-4x$	$-33ab$	$227a^2$	$-11b$
$-5x$	$42ab$	$-364a^2$	$5b$
$3x$	$-16ab$	$843a^2$	$-9b$
$8x$	$-27ab$	$-279a^2$	$13b$
$-2x$	$-38ab$	$-184a^2$	$-6b$

Subtract each from the other:

5. $5a^2 - 6ab + 3b^2$, and $4a^2 - 2ab + 7b^2$.

6. $13a^3 - 39a^2b + 39ab^2 - 13b^3$, and $5a^3 - 15a^2b + 15ab^2 - 5b^3$.

7. Multiply: a^4 by ab^3; $2x^3y$ by xy^3; ab^3 by a^2bc; $5a^2bc$ by $5a^2bc$.

8. Multiply $5a^3 + 7a^2b + 5ab^2 + 2b^3$ by $3a^2b$.

9. Multiply $x^2 + 2x + 1$ by $2ax$.

10. Divide $5a^3b^3 + 15a^2b^4 + 10a^4b^2$ by $5a^2b^2$.

11. Divide $6ax + 9a^2x^2 + 12a^3x^3 + 15a^4x^4$ by $3ax$.

12. Four times and five times a number equals 108; what is the number? *Ans.* 12.

13. One-sixth of a pole is in the mud, four-fifths in the water, and 1 foot above the water; how long is the pole?

14. The sum of two numbers is 26, and their difference is 8; what are the numbers?

15. The sum of two numbers is 8, and their difference is 26; what are the numbers?

16. John has 9 marbles more than Harry, and both together have 35; how many has each?

17. Take $7\,a$ and $-4\,a$, and by adding first a positive and then a negative quantity to each, illustrate the following principles: —

(1) Adding on a positive quantity gives a sum larger (or smaller by a negative quantity) than the quantity to which it is added.

(2) Adding on a negative quantity gives a sum smaller (or larger by a negative quantity) than the quantity to which it is added.

18. Make examples to illustrate the following principles: —

(1) Subtracting a positive quantity gives a remainder smaller (or larger by a negative quantity) than the minuend.

(2) Subtracting a negative quantity gives a remainder larger (or smaller by a negative quantity) than the minuend.

19. From 0 subtract: $-a^2x$; a^3b; $3\,ac$; $x-y$; $a-2\,b$; $a^2-2\,ab+b^2$.

LESSON X.*

Multiplication by Negative Quantities.

Multiply:

1. -7 by 3. Prove by addition.

2. 9 by -4. Prove by addition.

3. -5 by -3. Prove by addition.

4. $-8\,ax^2$ by -1.

5. $25\,a^3x^2$ by $-y$.

6. $-5\,x^2y^2$ by $-a$.

7. $-7\,a^5x^2$ by $-2\,a^2y$.

8. $-4a^2b^2$ by $2abc$. **9.** a^2-2ab by $-a$.

10. $-3x^2-2x+4$ by $-3x$.

11. $6a^2c-3ac^2$ by -2.

12. $-7x-9$ by -1.

13. $8x^2-2y^2-3$ by -1.

14. a^2+3a^2b+7 by $-2a^2x$.

15. $7a^2-3ab+2c^2-3b^2c$ by $5abc$.

16. $12x^3y+7x^2y^2+14xy^3$ by $2xy$.

17. $b^3-a^3-c^3$ by $-abc$.

18. $m^4+n^4-r^4-s^4$ by -1.

19. $a^4-a^3-a^2+a-1$ by $-a$.

20. $7+3x-6x^2+3x^3$ by $2x^2$.

21. From 0 subtract each of the following quantities: —

a; $5a$; $-7x$; -3 four times; $-2x$ three times; $3ab$; $27xy$; $-4ax$; $a-b$; $4-x$.

22. To 0 add each of the following quantities: —

6; -7; $5a$; $12xy$; $-6xy$; x four times; $-x$ four times; ab minus three times; $-ab$ minus three times; $6a^2x$ minus twice; $3a-b$ twice; $2x-3y$ minus once; $a-b-c$ minus once.

23. Three-eighths of a number, three-sevenths of it, and 11 added together equals the whole number; what is the number?

24. I bought the same number of lead-pencils, penholders, and slate-pencils. For the lead-pencils I paid 3 cents each, for the penholders, 5 cents each, and for the slate-pencils a cent each. The whole cost $1.08; how many did I buy of each?

25. A farmer sold a number of bushels of corn, twice as many bushels of wheat, and as many bushels of potatoes as of corn and wheat together. He got 40 cents a bushel for his corn, 80 cents a bushel for his wheat, and 50 cents a bushel for his potatoes. In all he received \$105.00; how many bushels did he sell of each?

26. From $18x^2 - 3x + 27$ subtract $12x^2 + 4x - 3$.

27. From $3a^3b^2 - 5ab^2 + 6b^3$ subtract $3a^3b^2 - 4a^2b^2 - 3ab^2 + 9b^3 + 7$.

28. From $a^3 - b^3$ subtract $a^3 - 3a^2b + 3ab^2 - b^3$.

LESSON XI.*

Multiplication continued — Powers of Quantities.

Multiply:

1. a^4 by a^3.
2. a^4 by $-a^3$.
3. $-a^4$ by $-a^3$.
4. a by a.
5. $-a$ by $-a$.
6. $-x^2$ by $-x^3$.
7. $6ax$ by $-2a^3$
8. $-3m$ by $2n$.
9. $-x^2$ by x.
10. $a^2 + 2ab + b^2$ by $a + b$.
11. $a^3 - 3a^2b + 3ab^2 - b^3$ by $a - b$.
12. $a^3 - 9a^2 + 27a - 27$ by $a - 3$.
13. $a + b + c$ by $a + b - c$.
14. $a - b - c$ by $a - b + c$.
15. $2x^2 - 8x + 7$ by $x + 3$.
16. $3x^3 - 9x^2 + 2x - 4$ by $x + 2$.
17. $a^2 + b^2$ by $a + b$.
18. $3x^3 + 4x - 5$ by $x - 6$.
19. $x + 7$ by $x - 7$.

20. $x + c$ by $x - c$.

21. $3a^3 - 7ad + 2c - 5$ by $a + c$.

Expand:

22. $(a - b)^3$. **23.** $(x + 2)^2$. **24.** $(x - 5)^3$.

Multiply:

25. $x^2 - 2bx + b^2$ by $x^2 + 2bx + b^2$.

26. $x^3 - a^3$ by $x^3 + a^3$.

27. A drover bought the same number of sheep and cows; for the sheep he paid \$3.00 each, and for the cows \$27.00; for all he paid \$270.00. How many did he buy of each? *Ans.* 9.

28. A drover bought a certain number of cows, the same number of oxen, and three times as many sheep; for the cows he paid \$25.00 each, for the oxen \$50.00 each, and for the sheep, \$3.00 each; it cost him \$7.00 to have them brought to market. The whole cost was \$679.00; how many did he buy of each?

29. Multiply $3a$ by $-2ax$, and subtract the product from zero.

30. Multiply $a^2 - b^2$ by $-a$, and subtract the product from zero.

31. Multiply $a + 5$ by $a - 5$, and subtract the product from zero.

LESSON XII.*

Division.

Divide:

1. a^4 by a^2.

2. a^4 by $-a^2$.

3. $-a^4$ by $-a^2$.

4. $-a^4$ by a^2.

5. a^3x by $-a^2x$

6. $-a^3b^2y$ by a^2by.

7. $-8c^3d$ by $-4c^3$.

8. $24x^2$ by $6x$.

9. $-18m^4$ by $3m^4$.

10. $-27ad$ by $-27a$.

11. a^2-ab by a.

12. $6x^3-6ax^2$ by $6x^2$.

13. $5a^2y-10ay$ by $-5ay$.

14. $14a^5b^3-7a^4b^2$ by $-7a^4b^2$.

15. $12ax-36a^2x$ by $12ax$.

16. $45b^8-30b^6+15b^4$ by $-15b^4$.

17. $8a^3b^2c-16a^4b^3c^2$ by $-8a^3$.

18. $-7a^5-14a^3$ by $-7a^3$.

19. $a-b+c$ by -1.

20. $7-4a$ by -1.

21. Take $a^2+4ab-3ac$; multiply by a^4b^2; then divide by a^5b; then multiply by $-b^3c^3$; then divide by $-bc^2$; what is the last quotient?

22. Multiply $a^2-2ab+b^2$ by -1, and then divide the product by -1.

23. Multiply $a-b+c-d$ by $-a$, and divide the result by -1.

24. Multiply $a^4-4a^3b+6a^2b^2-4ab^3+b^4$ by $a-b$, subtract $a^5-5a^4b+5ab^4-b^5$ from the product, and divide the remainder by $-10a^2b^2$.

25. Divide $-6a^2x^4+12a^3x^3-6a^4x^2$ by $-6a^2x^2$.

26. Divide $-ax^2y+bx^2y-cx^2y$ by $-x^2y$.

27. One-third, one-fifth, and one-ninth of a number equals 74 minus the number; what is the number?

28. 40 divided by a certain number gives 3 less than 64 divided by the same number; what is the number?

29. Multiply $a+b$ by $-a$ and subtract the product from 0.

30. From 0 subtract $-7a$; $6xy$; -36; $-8c$; -9; $3-x$; a^2-b^2.

31. Multiply $a-b$ by -1; by 1.

32. Divide $a-b$ by -1; by 1.

LESSON XIII.*

Division continued — Miscellaneous Examples.

Divide:

1. $-75a^3b^4$ by $-25a^2b^4$.

2. $125ax^2$ by $-25ax$.

3. x^2 by x.

4. x^2 by $-x$.

5. $96b^2c-72b^3c-36ab^2c$ by $-12b^2c$.

6. $a^3+3a^2b+3ab^2+b^3$ by $a+b$; by $-a-b$.

7. $a^3-3a^2b+3ab^2-b^3$ by $a-b$; by $-a+b$.

8. $x^3-3x^2y+3xy^2-y^3$ by $x^2-2xy+y^2$.

9. $x^4+4x^3y+6x^2y^2+4xy^3+y^4$ by $x^2+2xy+y^2$.

10. Divide the same quantity by $x+y$.

11. $3a^2+2a-8$ by $3a-4$.

12. x^2-5x+6 by $x-3$.

13. $6x^2-4x-42$ by $3x+7$.

14. $x^3+3x^2-25x+21$ by $x-3$.

15. $2a^2-3ax-2x^2$ by $2a+x$.

16. $6a^3+5a^2+9$ by $2a+3$.

17. $5x^3-23x^2+4$ by $5x+2$.

18. $4a^2b^2+4ab^2+b^2-c^2$ by $2ab+b+c$.

To find the value of x:

19. $-x = 8$. **20.** $2x - 5 = 4x - 31$.

21. A father is four times as old as his son, but in 14 years he will be only twice as old; what is the age of each? *Ans.* 7 and 28.

22. One man has worked three times as long as another, but if they continue to work 2 hours longer, he will have worked only twice as long; how long has each worked?

23. A man sold a certain number of horses at \$100 each, and bought two more than the same number of cows for \$30 each, and had \$640 left; how many horses did he sell, and how many cows did he buy?

24. Multiply $3a + 4b$ by itself.

25. Multiply $2x - 3y$ by itself.

LESSON XIV.*

Division continued — Square Root and Cube Root of Monomials — The Signs $\pm$ and $\sqrt{}$.

Divide:

1. $35a^3b$ by $7ab$.

2. $-15x^3$ by $-5x^2$.

3. $a^2 - b^2$ by $a - b$.

4. $a^3 - b^3$ by $a - b$.

5. $a^3 + b^3$ by $a + b$.

6. $a^4 - b^4$ by $a^2 - b^2$.

7. $a^4 - b^4$ by $a + b$; also by $a - b$.

8. $x^5 - y^5$ by $x - y$.

9. $x^5 + y^5$ by $x + y$.

10. $x^6 + y^6$ by $x^2 + y^2$.

11. Multiply $a^3 + b^3$ by $a - b$, and divide the product by $a^2 - b^2$.

12. Multiply $a^3 + b^3$ by $a^3 - b^3$, and divide the product by $a^2 - b^2$.

13. The square roots of 1; 4; 9; 16; 25; 36; 49; 64; 81.

14. Square the following quantities: x; $-x^2$; x^3; a^2; $-a^5$; b^6; c^2d^2; $-a^2b^3$.

15. Cube the following quantities: a; a^2; $-a^3$; x^2; x^4; $-ax^2$; $-a^2x^3$; a^2b^2c; $-ab$.

16. Square roots of the following quantities: x^2; x^4; b^6; a^2b^4; $a^2b^2c^2$; x^8y^4.

17. Cube roots of the following quantities: a^9; a^3; $-b^{12}$; $-a^3x^3$; $-a^6x^3$; a^9b^{12}.

18. Square the following quantities: $2a$; $-3a^2$; $-3a^2x$; $4a^2x^2$; $-5x$; $8a^2y^2$.

19. Cube the following quantities: -5; $3x$; $2x^2$; $-4ax$; $5a^2x^3$; $-7ax^4$.

20. Square root of the following quantities: 64; $4x^2$; $25a^4$; $81a^2b^4$; $144c^2d^4$.

21. Cube roots of the following: $8a^3$; $27a^9$; $-125a^3b^6$; $-64x^9y^6$; $343a^3x^{12}$.

22. Henry had eight marbles more than John; five times the number John had equalled three times the number Henry had; how many had each?

23. Divide 20 into two such parts that six times the less equals four times the greater.

24. One-third of a number less 5 equals one-fourth of it; what is the number?

LESSON XV.

Involution — Binomial Theorem.

Expand:

1. $(a^2)^3$.
2. $(x^3)^4$.
3. $(b^4)^5$.
4. $(-a^2)^2$.
5. $(-a^2)^3$.
6. $(-b^2)^4$.
7. $(-ab)^2$.
8. $(ab)^2$.
9. $(-ab)^2$.
10. $(ab)^2$.
11. $(2x^2y)^5$.
12. $(5x^3y^2)^3$.
13. $(-2x^3y)^4$.
14. $(2x^3y)^4$.
15. $(-c^3)^4$.
16. $(-3ac^3)^3$.
17. $(-x)^2$.
18. $(4x^2y^3)^5$.
19. $(m+n)^2$.
20. $(m+n)^3$.
21. $(m-n)^2$.
22. $(m-n)^3$.
23. $(a-x)^5$.
24. $(x+y)^7$.
25. $(a+2)^3$.
26. $(b-3)^4$.
27. $(x-3)^3$.
28. $(a+b)^3$.
29. $(c+5)^2$.
30. $(c-6)^2$.

NOTE. For advanced work, see Lesson XXXII.

31. Five times the sum of a certain number and six, equals four times the sum of the same number and five; what is the number?

32. A farmer had a certain number of sheep, and his neighbor had ten more; the former sold his for \$4.00 each, and the latter for \$3.00, and each received the same sum for all; how many sheep had they?

33. Four-sevenths and three-fifths of a certain time added together equals the whole time plus 6 hours; how long is the time?

Divide:

34. $x^3 + 6x^2 - x - 30$ by $x + 5$.

35. $a^3 + a^2 - 9a - 9$ by $a + 1$.

36. $x^3 - 16x^2 - 3 + 48$ by $x^2 - 16$.

37. $x^4 - 9x^3 + 4x - 36$ by $x^3 + 9$.

Quick oral work: —

Expand: $(a+b)^2$; $(a+b)^3$; $(a-b)^3$; $(a-b)^4$; $(a+b)^4$; $(a+b)^5$; $(a-b)^5$.

$(x+y)^2$; $(x-y)^2$; $(x+y)^3$; $(x-y)^3$; $(x-y)^4$; $(x+y)^4$; $(x-y)^5$; $(x+y)^5$; $(x-y)^7$.

LESSON XVI.*

Relation of Quantities — The Parenthesis — Factoring.

ORAL EXERCISE.

Give the relation of:

1. cd to c and d.

2. $m + n$ to m and n.

3. $m - n$ to m and n.

4. $2xy$ to x and y.

5. 9 to 3; to -3.

6. $16a^2$ to $4a$; to $-4a$.

7. $3a$ to $9a^2$.

8. $25a^2b^4$ to $5ab^2$; to $-5ab^2$.

9. $14a$ to 7 and a.

10. $4a$ to a and $3a$.

11. $-5a$ to $-2a$ and $-3a$.

12. $3c$ to $-5c$ and $8c$.

13. $-3c$ to $-7c$ and $4c$.

14. $-5x$ to $-12x$ and $7x$.

15. $-12x^2$ to $4x$ and $-3x$.

16. $27x^2$ to $3x$ and $9x$.

17. $-x$ to $8x$ and $-9x$.

18. x to $7x$ and $-6x$.

19. $3x$ to $7x$ and $4x$.

20. a^2x^2 to ax; to $-ax$.

21. $7a^2x$ to $9a^2x$ and $-2a^2x$.

22. $-18a^4x^2$ to $9a^2x$ and $-2a^2x$.

23. $-45a^3b$ to $15a^3$ and $-3b$.

24. $18ax^2$ to $9ax$ and $2x$.

25. $8b$ to 4 and b.

26. $10x$ to 5 and x.

Factor:

27. $a^3x + a^2y - a^3$.

28. $6x^2y - 12xy^2$.

29. $5x^3y^2 + 10x^2y^3 - 5x^2y^2$.

30. $a^3 + a^2b + a^2c$.

31. $7ax^2 + 14ax$.

32. $18 - 27ab + 9a$.

33. $7a - 14ax + 21ax^2$.

34. $15a^2b^3 - 10a^3b^2 + 25a^2b^2$.

35. $6a - 12a^2 + 18a^3$.

Combine:

36. $(x - 1)(x + 1)$.

37. $(x + 2a)(x - 2a)$.

38. $(2ax + 3by)(2ax - 3by)$.

39. $(3a^2b - 5ac)(3a^2b + 5ac)$.

40. $(a^2 + b^2)(a^2 - b^2)$.

Factor:

41. $c^2 - d^2$.

42. $m^4 - n^2$.

43. $4\,a^2 - 9\,b^2$.

44. $25\,m^2 - 16\,n^2$.

45. $81\,a^2b^2 - 36$.

46. $625\,a^6 - b^2$.

47. $64\,b^4 - 4$.

48. $4\,a^2 - 1$.

49. $1 - 9\,b^2$.

50. $a^4b^2 - 1$.

Separate into three factors:

51. $ax^2 - ay^2$.

52. $5\,a^2b^2 - 5\,x^2y^2$.

53. $x^4 - y^4$.

54. $16\,x^4 - 81\,y^4$.

Add:

55. $a\,(a - 5)$ to $5\,(a - 5)$.

56. $2\,x\,(x + 6)$ to $7\,(x + 6)$.

Subtract:

57. $6\,(m + 6)$ from $m\,(m + 6)$.

58. $b\,(a + b)$ from $a\,(a + b)$.

59. If a number be subtracted from 25, and also 14 be subtracted from the number, five times the first remainder equals six times the second; what is the number?

60. John and Henry had the same number of marbles; John gained 7 and Henry lost 8, then John had twice as many as Henry; how many had each in the beginning?

ORAL EXERCISE.

Combine: $(x + 3)\,(x - 3)$; $(a - 7)\,(a + 7)$; $(3\,a - 5\,b)\,(3\,a + 5\,b)$; $(2\,xy - 3\,a)\,(2\,xy + 3\,a)$; $(2\,x^3 - 2\,y)\,(2\,x^2 + 2\,y)$; $(5\,x^3 - ab)\,(5\,x^3 + ab)$.

Expand: $(x + 3)^2$; $(x + 2)^2$; $(x - 4)^2$; $(x - 7)^2$.

LESSON XVII.

The Parenthesis — Problems to be Solved by Inspection.

Remove parentheses:

1. $a + (a - b)$.
2. $a - (a - b)$.
3. $9 + (x - 4)$.
4. $9 - (x - 4)$.
5. $x + (a - b + c)$.
6. $a^3 - (a^2 - a + 1)$.
7. $12 - (7 - a + c)$.
8. $15 - (a^2 - a^3 + 15)$.

Remove parentheses and simplify:

9. $6a^2 + 3a - 2(b - c + d)$.
10. $9a^2 + 5a - 2(a^2 + 2a)$.
11. $5x^2 + 9ax - 4x - 4x(x + 2a - 1)$.
12. $8a^3 + 7a^2 - 8(a^3 - a^2 + 1)$.
13. $2x^2 + (3x^2 - x) - 4(x^2 - 2x - 1)$.
14. $a^3 - 2(a^2 - 3a + 2) + 2(2a + 2)$.
15. $a^3 - (a^2 + a) + 2(a^3 + a^2) - 3(a + 1)$.

Combine by inspection:

16. $(3a - 2x)(3a + 2x)$.
17. $(a^2 - b^2)(a^2 + b^2)$.
18. $(5a - 3b)(5a + 3b)$.
19. $(2ax + 3by)(2ax - 3by)$.
20. $(6x^2 + 5y^2)(6x^2 - 5y^2)$.
21. $(2a^2b^2 - 5)(2a^2b^2 + 5)$.
22. $(x + 1)(x - 1)$.
23. $(xy - 1)(xy + 1)$.
24. $(1 + a)(1 - a)$.

Factor:

25. $a^2 - y^2$.

26. $4a^2 - 9$.

27. $25 - 9a^2b^2$.

28. $3x^2 - 12y^2$.

29. $64a^2x^2 - 36b^2x^2$.

30. $16a^4 - 81b^4$ (three factors).

31. $25m^2 - 16n^2$.

32. $625a^4 - 9b^4$.

33. $a^4 - b^2$.

34. $49a^6 - 4b^2$.

35. $81a^4 - 36a^2$.

36. $16a^2 - 1$.

37. $1 - 4x^2$.

38. $1 - 9x^4$.

39. $25x^6 - 1$.

40. $x^4 - 1$.

41. $1 - x^4$.

42. $625a^2b^2 - 4x^2y^2$.

43. $9a^4x^4 - 4b^2y^2$.

44. $a^4b^2 - 4$.

Expand:

45. $(x + y)^4$.

46. $(x - y)^4$.

47. $(x + 3)^3$.

48. $(x - 4)^3$.

49. $(x - 5)^3$.

50. $(x + 3)^2$.

Factor together the two first terms and the two last terms, and simplify:

51. $ax + bx - ay - by$. *Ans.* $(x - y)(a + b)$.

52. $3ab + 3ac + 4bx + 4cx$.

Ans. $(3a + 4x)(b + c)$.

53. $10 - 5a - 6b + 3ab$.

54. $3ax - xb + 12a - 4b$.

55. $2ab + 3bc - 10a - 15c$.

56. $6ax - 10ay - 9bx + 15by$.

Ans. $(3x - 5y)(2a - 3b)$.

LESSON XVIII.

Multiplication at Sight — Factoring — Problems in which Two Letters are Used to Indicate the Unknown Quantities.

Expand at sight:

1. $(x + 2)^2$.
2. $(x + 7)^2$.
3. $(x + 4a)^2$.
4. $(x + 3c)^2$.
5. $(a + 3b)^2$.
6. $(a + 5b)^2$.
7. $(x + 1)^2$.
8. $(x + 9)^2$.

Factor by inspection:

9. $x^2 + 14x + 49$. *Ans.* $(x + 7)^2$.
10. $x^2 + 20x + 100$. *Ans.* $(x + 10)^2$.
11. $x^2 + 4ax + 4a^2$. *Ans.* $(x + 2a)^2$.
12. $x^2 + 6ax + 9a^2$.
13. $x^2 + 8x + 16$.
14. $a^2 + 2a + 1$.
15. $b^2 + 10bx + 25x^2$.

Expand at sight:

16. $(x - 7)^2$.
17. $(y - 3)^2$.
18. $(a - 3b)^2$.
19. $(b - 15)^2$.
20. $(x - 3a)^2$.

Factor by inspection:

21. $x^2 - 18x + 81$. *Ans.* $(x - 9)^2$.
22. $x^2 - 12x + 36$.
23. $x^2 - 24x + 144$.
24. $a^2 - 30a + 225$.
25. $a^2 - 22a + 121$.

Multiply at sight:

26. $x + 1$ by $x + 4$. *Ans.* $x^2 + 5x + 4$.
27. $x - 3$ by $x - 5$. *Ans.* $x^2 - 8x + 15$.

28. $x + 1$ by $x + 2$.
29. $x + 2$ by $x + 5$.
30. $x + 8$ by $x + 3$.
31. $x - 4$ by $x - 6$.
32. $x - 2$ by $x - 4$.
33. $x - 7$ by $x - 1$.
34. $x + 2a$ by $x + 5a$.
35. $x + 3a$ by $x + 7a$.
36. $x - 4a$ by $x - a$.

Factor by inspection:

37. $x^2 + 4x + 3$.
38. $a^2 - 5a + 6$.
39. $x^2 + 3x + 2$.
40. $x^2 - 10x + 16$.
41. $x^2 + 12x + 11$.
42. $x^2 + 5x + 4$.
43. $x^2 - 4x + 3$.

Multiply:

44. $5 + x$ by $7 + x$.
45. $2 + y$ by $5 + y$.

Factor:

46. $15 + 8x + x^2$.
47. $12 + 7x + x^2$.

ORAL EXERCISE.

48. Find two numbers:

1. Whose sum is 5, and product 6.
2. Whose sum is 4, and product 3.
3. Whose sum is -8, and product 15.
4. Whose sum is -3, and product -10.
5. Whose sum is 4, and product -21.
6. Whose sum is 12, and product 35.
7. Whose sum is -12, and product 45.
8. Whose sum is $7a$, and product $-30a^2$.
9. Whose sum is $-8b$, and product $-20b^2$.
10. Whose sum is $-15c$, and product $56c^2$.
11. Whose sum is 7, and product 12.
12. Whose sum is -7, and product -44.

49. The sum of two numbers is 13 and the difference 3; what are the numbers?

50. Four times one number plus twice a second is 34, and twice the sum of the two is 22; what are the numbers?

51. Five times a number plus three times a second is 41, and three times the first minus three times the second is 15; what are the numbers?

LESSON XIX.

Factoring Arranged by Cases.

CASE I. General problem: $ab \pm ac = a\,(b \pm c)$.

Factor:

1. $a^3b - 3\,a^2c$.

2. $7\,abx + 14\,aby$.

3. $5\,x^2 + 10\,x^3$.

4. $25\,a^2b^3 - 5\,a^2b$.

5. $13\,a^2 + 13\,a$.

6. $9\,a^2b + 18\,a^2c - 27\,a^2d$.

7. What are the peculiar features of Case I.?

CASE II. General problem: $a^2 - b^2 = (a + b)\,(a - b)$.

Factor:

1. $4 - 9\,a^2$.

2. $25\,a^2 - 16\,b^2$.

3. $64\,a^2x^2 - 100\,b^2y^2$.

4. $1 - 4\,x^2$.

5. $81\,y^4 - 1$. (3 factors.)

6. $121 - 4\,a^2b^2c^2$.

7. What are the peculiar features of Case II.?

CASE III. General problem: $a^2 + 2\,ab + b^2 = (a + b)\,(a + b)$.

Factor:

1. $m^2 + 2mn + n^2$.
2. $4m^2 + 12mn + 9n^2$.
3. $64b^2 + 32bx + 4x^2$.
4. $25a^2b^2 + 20abx + 4x^2$.
5. $16x^2 + 8x + 1$.
6. $1 + 4x^2 + 4x^4$.
7. What are the peculiar features of Case III.?

CASE IV. General problem: $a^2 - 2ab + b^2 = (a - b)(a - b)$.

Factor:

1. $36a^2 - 36ab + 9b^2$.
2. $x^2 - 10x + 25$.
3. $49 - 28c + 4c^2$.
4. $100x^2y^2 - 100xy + 25$.
5. $5a^2 - 10ab + 5b^2$. (3 factors.)
6. $81x^4 - 72xy + 16y^2$. (4 factors.)
7. What are the peculiar features of Case IV.?

CASE V. General problem: $(a + b)(a + c) = a^2 + (b + c)a + bc$.

Factor:

1. $x^2 + 5x + 6$.
2. $y^2 + 3y + 2$.
3. $m^2 + 13m + 40$.
4. $b^2 + 15b + 54$.
5. $7x^2 + 56x + 105$. (3 factors.)
6. $3c^2 + 18c + 24$.
7. What are the peculiar features of Case V.?

CASE VI. General problem: $(a - b)(a - c) = a^2 - (b + c)a + bc$.

Factor:

1. $x^2 - 7x + 6$.
2. $a^2 - 11x + 10$.
3. $x^2 - 17x + 30$.
4. $x^4 - 12x^2 + 32$.

5. $x^4 - 9x^2 + 20$. 6. $x^2 - 8ax + 15a^2$.

7. What are the peculiar features of Case VI.?

Combine by inspection:

1. $(a + 9)(a - 8)$. 3. $(x + 2a)(x - 4a)$.

2. $(x + 4)(x - 8)$. 4. $(x + 8b)(x - b)$.

Case VII. General problem: $(a + b)(a - c) = a^2 + (b - c)a - bc$.

Factor:

1. $x^2 + 3x - 10$. 6. $a^2 - 2a - 63$.

2. $y^2 - 5y - 14$. 7. $a^2 + 2a - 63$.

3. $x^2 - 11x - 12$. 8. $x^2 - 3x - 40$.

4. $a^2 + a - 56$. 9. $x^2 - 3ax - 10a^2$.

5. $a^2 - a - 56$. 10. $x^2 + 3cx - 4c^2$.

What are the peculiar features of Case VII.?

For Cases VIII. and IX., see Lesson XXI.

1. A and B have herds of cattle of which A's is the larger; the number in both herds is 95, and three times the number A has minus the number B has is 117; how many had each? *Ans.* A, 53; B, 42.

2. $\frac{x}{3} + \frac{y}{2} = 17$. $\frac{x}{4} + \frac{y}{3} = 12$.

LESSON XX.*

Common Divisor — Greatest Common Divisor, G. C. D.

In the following examples find two common divisors, one of which will be the greatest (integral) divisor pos-

sible. Also quotients, when the quantities are divided by it.

1. 36, 48, 72, 96.
2. $5\,a^2b$, $10\,a^3b^2$, $15\,a^2b^2x$.
3. $9\,x^2y$, $27\,xy$, $45\,x^3y^3$.
4. a^2c, a^2d, a^3, $5\,a^2$.
5. a^2x^2y, $a^2x^2 - 5\,a^3x$, $abx + acx$.
6. $x^2 - y^2$, $ax - ay$, $x^2 - 2\,xy + y^2$.
7. 6, 7, 5, 11.

Find G. C. D. only:

8. $7\,a + 7\,b$, $ax + ab$, $a^2 - b^2$.
9. $12\,b - 24$, $bx - 2\,x$, $x^2 - 4$.
10. $a^2x - 3\,a^3$, $3\,ax^2 - 27\,a$, $ax - 3\,a$.
11. $x^2 - y^2$, $x^2 - 2\,xy + y^2$, $ax^2 - axy$.
12. $3\,a^2 - 3\,c^2$, $a^2 + 2\,ac + c^2$, $5\,ab + 5\,bc$.
13. $m^2 - 16$, $m^2 + 8\,m + 16$, $mx + 4\,x$.
14. $36 - a^2$, $36 + 12\,a + a^2$.
15. $81 - a^4$, $9 + 6\,a + a^2$.
16. $x^2 - 25$, $x^2 + 8\,x + 15$.
17. $x^2 - 49$, $x^2 - 9\,x + 14$, $3\,x + 21$.
18. $y^2 + 6\,y + 9$, $y^2 - 9$, $y^2 + 2\,y - 3$.
19. $a^2 - 3\,a - 10$, $a^2 - 10\,a + 25$.
20. $9\,a^2 - 25\,b^2$, $15\,ax - 25\,bx$.
21. $c^2 - 8\,c + 15$, $c^2 - c - 20$, $2\,c - 10$.

Find the value of x or y:

22. $\frac{1}{4} - \frac{1}{5} = \frac{1}{x}$. *Ans.* $x = 20$.
23. $\frac{1}{3} + \frac{1}{5} = \frac{1}{x}$. *Ans.* $x = 1\frac{7}{8}$.
24. $\frac{1}{3} + \frac{2}{5} = \frac{1}{y}$.
25. $\frac{x}{4} + \frac{x}{5} = 9$.

26. One man can do a piece of work in 3 hours and another in 6 hours; how long will it take them both to do it?

27. Two pipes discharge water into a cistern; one will fill it in 8 hours and the other in 4 hours; how long will it take both to fill it?

28. Of two pipes one will fill a tank in 4 hours, and both in 3 hours; how long will it take the other to fill it?

29. In a school of boys and girls, two times the number of boys plus five times the number of girls equals 100, and four times the number of boys minus five times the number of girls equals -10; how many are there of each?

30. Divide $a^7 + b^7$ by $a + b$.

31. Multiply $a^4 - a^3b + a^2b^2 - ab^3 + b^4$ by $a + b$.

32. $m + (n - r + 3)$; remove parentheses.

LESSON XXI.

Cases of Factoring, Continued — G. C. D.

Divide:

1. $x^3 + y^3$ by $x + y$.

2. $x^3 - y^3$ by $x - y$.

3. $y^3 + 8$ by $y + 2$.

4. $y^3 - 8$ by $y - 2$.

5. $x^5 + y^5$ by $x + y$.

6. $x^5 - y^5$ by $x - y$.

FACTORING. CASE VIII. General Problems.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$.

Factor:

1. $m^3 + n^3$. 3. $(a^2)^3 + (b^2)^3$. 5. $x^3 + 125$.
2. $c^3 + d^3$. 4. $a^3 + 27$. 6. $x^6 + 64$.
7. What are the peculiar features of Case VIII.?

CASE IX. General Problems.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$
$$a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4).$$

Factor:

1. $r^3 - s^3$. 3. $(m^2)^5 - (n^2)^5$. 5. $125 - a^3$.
2. $c^3 - m^3$. 4. $x^3 - 343$. 6. $27 - b^6$.
7. What are the peculiar features of Case IX.?

For Case X., see Lesson XXIII.

Find the greatest common divisor (G.C.D.) of the quantities in each of the following:

1. a^2b, ab^2, and $-a^2b^2$.
2. $3xy$, $9ax$, $-3bx^2$, and $12cx$.
3. $7ax + 7bx$ and $3a^2 - 3b^2$.
4. $ax - ay$, $x^2 - y^2$, and $x^2 - 2xy + y^2$.
5. $a^2 + 2ab + b^2$ and $a^2 - b^2$.
6. $a^2 - b^2$ and $a^3 + b^3$.
7. $x^2 - 4$ and $x^2 - 4x + 4$.
8. $9 - a^2$ and $9 - 6a + a^2$.
9. $x^2 - a^2$, $x^2 + (a + b)x + ab$, and $5x + 5a$.
10. $x^2 - 4x + 4$ and $x^2 + x - 6$.

Problems in division.

1. Divide $(x + y)^3$ by $(x + y)^2$.
2. Divide $(a + b)^5$ by $(a + b)^3$ and expand the quotient.

3. Divide $(x+3)^5$ by $(x+3)^2$ and expand the quotient.

4. The G. C. D. of $(c+d)^4$, $(c+d)^3$, and $(c+d)^5$.

Miscellaneous examples for factoring.

1. x^2+5x+4.
2. $x^2-12x+32$.
3. $x^2-3x-108$.
4. $x^2-x-132$.
5. x^2+x-72.
6. $x^2-2x-66$.
7. $25x^2-9c^2$.

LESSON XXII.*

Fractions — Use of the G. C. D. — Axioms.

REDUCE to lowest terms:

1. $\frac{3ax^2}{6ax}$.
2. $\frac{16a^3b^2}{8a^4b^3}$.
3. $-\frac{5ax^3y^2}{25ax^4y^2}$.
4. $\frac{18m^2y^2}{9m^2y^2}$.
5. $-\frac{17a^2b^2}{34a^3b^3}$.
6. $\frac{39x^4y^4}{13x^2y^4}$.
7. $\frac{a^3b+a^4}{3a^3}$.
8. $\frac{a^3b+a^4}{a+b}$.
9. $\frac{3x^3-3x^2y}{6ax^2}$.
10. $\frac{ax^3+ax^2y}{ax+ay}$.
11. $\frac{x^2-y^2}{x^2-2xy+y^2}$.
12. $\frac{a^2+2ab+b^2}{a^2-b^2}$.
13. $\frac{a^3+b^3}{a^2-b^2}$.
14. $\frac{4a(x^3-y^3)}{8a^2(x^2-y^2)}$.

15. $\dfrac{x^2+7x+12}{x^2+9x+20}$.

16. $\dfrac{a^2-6a+5}{a^2-3a-10}$.

17. $\dfrac{x^2-16}{x^2+8x+16}$.

18. $\dfrac{x^3-27}{x^2-6x+9}$.

19. Give the axiom by which you find the value of x in each of the following: —

1. $x+6=9$.
2. $x-3=7$.
3. $3x=15$.
4. $\dfrac{x}{5}=2$.

20. $2x+3y=35$.
$3x+y=35$.

21. $3x+y=33$.
$4x-3y=57$.

What axioms have you used in solving the above?

Use two letters for the unknown quantities in the following: —

22. A cistern holding 64 bbls. of water, is filled by two pipes; both running 8 hours will fill it, or No. 1 running 7 hours and No. 2 running 5 hours will fill it; how many bbls. per hour does each discharge into the cistern?

Interpret the negative result.

23. A and B are together worth \$3000; but twice A's property added to B's would be \$6500; what is each worth?

Find the value of x in terms of a, b, and c:

24. $x+b=a$.

25. $x-ab=a^2$.

26. $ax+ab=a^2$.

27. $ax+b=c$.

28. $\dfrac{x}{b}+b=\dfrac{a^2}{b}$.

29. $ax+bx=c$.

30. Expand $(x+b)^9$.

31. Expand $(x-c)^{10}$.

32. Expand $(2x-5b)^8$.

Factor:

33. $81\,x^2 - 25\,y^2$.

34. $x^2 + 16\,ax + 64\,a^2$.

35. $x^2 - 18\,abx + 81\,a^2b^2$.

36. $x^2 - 18\,x - 17$.

37. $a^2 + 17\,a - 30$.

LESSON XXIII.*

Least Common Multiple, L. C. M. — Factoring, Case X.

In each of the following examples find two common multiples, one of which will be the least common multiple, L. C. M.

1. 7, 21, 35, and 15. Prove.

2. a^2, ab^2, a^2c^2, b^2c^2. Prove.

3. x^2, y^2, $3\,x$, $5\,y$.

4. $2\,x$, $3\,y$, $2\,y\,(a + b)$.

5. $5\,m$, $3\,mn$, $5\,n$.

Find the least common multiple:

6. $3\,a$, $4\,ax$, $2\,a^2x^2$, $12\,a^2x$. Prove.

7. $16\,am$, $-8\,m^2$, $12\,a^2$, $-3\,a^2m^2$.

8. $a + b$, $a - b$.

9. $x + 3$, $x - 3$, $x^2 - 9$.

10. a^2, $b + 2$, $b^2 - 4$.

11. $3\,ab$, $3\,(a + b)$, $2\,(a - b)$.

12. $a^2 - b^2$, $a^2 - 2\,ab + b^2$, $a\,(a - b)$.

13. $a^2 - b^2$, $a^2 + 2\,ab + b^2$, $a^2 + ab$.

14. $x + 3$, $x - 3$, $x^2 - 9$.

15. $x + 2$, $x - 5$, $x^2 - 3\,x - 10$.

16. $x^2 + 4\,x + 3$, $x^2 + x - 6$, $ax + 3\,a$.

17. $x^2 - 25$, $x^2 + 2x - 35$, $ax - 5a$.

18. $x^2 - 2x - 8$, $x^2 - 4$, $x^2 - 6x + 8$.

19. $a^2 - b^2$, $a^3 - b^3$, $a^2 - ab$.

20. $a^2 - b^2$, $a^3 + b^3$, $a^2 + 2ab + b^2$.

Add the following:

21. $5(x + 6)$, $4(x + 6)$, $-3(x + 6)$, $-7(x + 6)$.

22. $3(a + b)$, $-4(a + b)$, $5(a + b)$, $9(a + b)$, $-2(a + b)$.

23. $a(c + d)$, $b(c + d)$. **24.** $a(x - y)$, $-b(x - y)$.

25. $a(a - b)$, $-b(a - b)$.

26. $a(a + b - 3)$, $b(a + b - 3)$.

FACTORING. CASE X. General Problems.

$ac + ad + bc + bd = a(c + d) + b(c + d) = (a + b)(c + d)$.

$ac - ad + bc - bd = a(c - d) + b(c - d) = (a + b)(c - d)$.

$ac - ad - bc + bd = a(c - d) - b(c - d) = (a - b)(c - d)$.

Factor:

27. $ax + ay + dx + dy$. **28.** $ax - 3a + 5x - 15$.

29. $a^3 - 3a^2 - 7a + 21$.

30. $6ax + 9ay - 4bx - 6by$.

31. $am - 2bm - an + 2bn$.

32. $a^2c^2 - abc^2 - a^2bc + ab^2c$.

NOTE. For Case XI., see Lesson XXXV.

Find value of x.

33. $x - (60 - x) = 20$. **34.** $2x - (x - 7) = 22$.

35. $x - (8 - 2x) = 1$.

36. $\frac{1}{2}+\frac{1}{8}=\frac{5}{x}$.

37. $\frac{1}{a}+\frac{1}{b}=\frac{1}{x}$.

38. A can mow a field in 4 hours, and B in 3 hours; how long will it take both? *Ans.* 1⅐ hours.

39. A and B together can mow a field in 5 hours, and A alone can do it in 8 hours; how long will it take B to do it?

Reduce to lowest terms:

40. $\frac{x^2-y^2}{x^3+y^3}$.

41. $\frac{x^2-xy}{x^3-y^3}$.

42. $\frac{x^2-x-42}{x^2+x-56}$.

43. $\frac{a^2-2a+1}{a^2+5a-6}$.

LESSON XXIV.*

Use of the L. C. M. — Addition and Subtraction of Fractions.

CHANGE the following fractions to a common denominator:

1. $\frac{a}{3}$ and $\frac{a}{4}$.

2. $\frac{a}{b}$ and $\frac{a}{c}$.

3. $\frac{a+b}{a}$ and $\frac{a+b}{a-b}$.

4. $\frac{a}{a+b}$ and $\frac{b}{a-b}$.

Add the following:

5. $\frac{x}{2}, \frac{x}{3}$.

6. $\frac{x}{a}, \frac{x}{b}$.

7. $\frac{a+2}{3}, \frac{a+5}{4}$.

8. $\frac{x+5}{5}, \frac{x-3}{3}$.

9. $\frac{a+b}{a}, \frac{a+b}{b}$.

10. $\frac{x+6}{3}, \frac{x-7}{4}$.

11. $\frac{7}{a+b}, \frac{7}{a}$.

12. $\frac{3}{x+2}, \frac{4}{x}$.

13. $\frac{a+b}{a-b}, \frac{a-b}{a+b}$.

14. $\frac{x+2}{x-2}, \frac{x-2}{x+2}$.

15. $\frac{x+3}{x+1}, \frac{x-6}{x-1}$.

Subtract:

16. Quantities as in 13, second from first.

17. Quantities as in 14, second from first.

18. Quantities as in 15, second from first.

19. From $\frac{x}{a}$ take $\frac{x}{b}$.

20. From $\frac{a}{b}$ take $\frac{b}{a}$.

21. From $\frac{x}{3}$ take $\frac{x}{5}$.

22. From $\frac{3x+7}{3}$ take $\frac{4x+9}{4}$.

23. From $\frac{2a+8}{5}$ take $\frac{3a-7}{6}$.

24. From $\frac{a+4}{5}$ take $\frac{a+3}{6}$.

25. From $\frac{a+b}{3}$ take $\frac{a+b}{6}$.

26. From $\frac{a+b}{a}$ take $\frac{a+b}{b}$.

27. From $\frac{a+b}{a-b}$ take $\frac{a-b}{a+b}$.

28. From $\frac{a^2+b^2}{a^2-b^2}$ take $\frac{a-b}{a+b}$.

Unite the following as indicated by signs:

29. $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$.

30. $\frac{a+b}{a-b} + \frac{a-b}{a+b} - \frac{a^2+b^2}{a^2-b^2}$.

31. $\frac{3+2x}{2-x} - \frac{2-3x}{2+x} + \frac{16x-x^2}{4-x^2}$.

32. $\frac{1}{a+b} + \frac{b}{a^2-b^2} - \frac{a}{a^2+b^2}$.

33.
$$\frac{a+b}{(b-c)(c-a)} + \frac{b+c}{(c-a)(a-b)} + \frac{c+a}{(a-b)(b-c)}.$$

34.
$$\frac{1}{(x+y)(x+z)} - \frac{1}{(y+z)(x+y)} + \frac{1}{(x+z)(y+z)}.$$

35.
$$\frac{ac}{(a-b)(b-c)} + \frac{bc}{(a-b)(c-a)} + \frac{ab}{(b-c)(c-a)}.$$

36.
$$\frac{c+a}{(a-b)(a-c)} + \frac{a+b}{(b-c)(b-a)} + \frac{b+c}{(c-a)(c-b)}.$$

Find the value of x:

37. $3x - (100 + x) = 0$.

38. $ax - (bx + c) = a - b - c$. *Ans.* $x = 1$.

39. The sum of two quantities is a and the difference b; what are the quantities?

40. A grocer has tea of two prices; 5 lbs. of the higher priced and 4 of the lower are worth \$6.27; 2 lbs. of each are worth \$1.76; find the price of each kind.

Reduce to lowest terms. See Case X., factoring.

41. $\frac{2ab+5b+6a+15}{b^2+4b+3}$. *Ans.* $\frac{2a+5}{b+1}$.

42. $\frac{ax+3a-bx-3b}{a^2-2ab+b^2}$.

43. $\frac{3ax-ay-6x+2y}{a^2+2a-8}$.

LESSON XXV.*

Mixed Numbers to Fractions — Fractions to Mixed Numbers.

CHANGE to a fraction:

1. $1+\frac{b}{a}$.
2. $3a+\frac{3b^2}{a}$.
3. $2x-\frac{a}{b}$.
4. $2x-a+\frac{a}{3}$.
5. $4a+\frac{ab}{a+b}$.
6. $a+b-\frac{a^2+b^2}{a+b}$.
7. $a-b+\frac{4ab}{a-b}$.
8. $x+3+\frac{9}{x-3}$.
9. $x-5+\frac{25}{x+5}$.
10. $a+b-\frac{a^2+b^2}{a-b}$.
11. $x-8+\frac{15x}{x-7}$.
12. $x+3-\frac{x+12}{x-4}$.

Change to a mixed number:

13. $\frac{x^2+9x+17}{x}$.
14. $\frac{2a^2-4a+8}{2a}$.
15. $\frac{a^2-ab+b^2}{a}$.
16. $\frac{3x^3-7x^2+4x+3}{x-2}$.

17. $\frac{5x^2+12x-41}{x^2+6}$. 18. $\frac{a^2+b^2}{a+b}$.

19. $\frac{a^2+ab+b^2}{a+b}$.

20. $\frac{15x^3+36x^2+8x-3}{5x+2}$.

21. $\frac{6a^3-2a^2+2a-7}{3a^2+2a+3}$.

Add:

22. $\frac{a}{a+b}$ and $\frac{b}{a+b}$. 23. $\frac{x}{x+3}$ and $\frac{x}{x-3}$.

24. $\frac{7}{a+b}$ and $\frac{4}{a^2-2ab+b^2}$.

25. Subtract $\frac{a}{a+2}$ from $\frac{a}{a+1}$.

26. Subtract $\frac{a-b}{a}$ from $\frac{a-b}{b}$.

Reduce to lowest terms:

27. $\frac{c^2-d^2}{c^2-2cd+d^2}$. 28. $\frac{x^2+7x+12}{x^2-3x-28}$.

29. $\frac{5a^3+10a^2b}{5(a^4-b^4)}$.

30. A farmer had b sheep; he lost a third of them, and then he lost a third of the remainder; how many did he have left?

31. Henry had $5a$ marbles; he lost half of them, and then he lost $\frac{3a}{2}$; how many had he left?

32. Simplify:

$$\frac{1}{(a-b)(a-c)}+\frac{1}{(a-c)(b-c)}+\frac{1}{(b-c)(a-b)}.$$

LESSON XXVI.*

Multiplication and Division of Fractions by Whole Quantities.

Multiply:

1. $\frac{a}{b}$ by b.
2. $\frac{a^2b}{3c^2}$ by $3ac$.
3. $\frac{2x^2y}{3am}$ by $3my$.
4. $\frac{x^2y^2}{5ab}$ by $10xy$.
5. $\frac{a}{b}$ by $3ax$.
6. $\frac{7a^3b^2}{9c^2d}$ by $18bc$.
7. $\frac{3a}{a+b}$ by $2b$.
8. $\frac{xy}{x-y}$ by xy.
9. $\frac{ab}{a-b}$ by $2(a-b)$.
10. $\frac{3ax}{2(x+y)}$ by x^2-y^2.
11. $\frac{a-b}{a+b}$ by a^2-b^2.
12. $\frac{x+5}{x^2-5x+6}$ by $x^2-7x+10$.
13. $\frac{x+a}{x^2+3ax-28a^2}$ by $x^2-5ax+4a^2$.

Divide:

14. $\frac{a}{b}$ by a.
15. $\frac{x^3y}{2a}$ by $3x^2a$.
16. $\frac{3ab^2}{2c}$ by $3abc$.
17. $\frac{5a^2b}{4c^2}$ by $5a^3b^2$.
18. $\frac{x}{5}$ by $2x$.
19. $\frac{12a^2c^2}{bm}$ by $4ac^2m$.
20. $\frac{a+b}{a}$ by $a(a+b)$.
21. $\frac{(x-y)^2}{xy}$ by $2(x-y)$.
22. $\frac{ab}{a-b}$ by $a(a+b)$.
23. $\frac{a(x-y)}{b}$ by $b(x-y)$.

24. $\frac{a^2 - 2ab + b^2}{c}$ by $a^2 - b^2$.

25. $\frac{a^2 + 2ab + b^2}{5}$ by $2(a^2 - b^2)$.

26. $\frac{a^3 + b^3}{a - b}$ by $a^2 - b^2$.

27. $\frac{x^2 - 2x - 35}{x + 2}$ by $x^2 + 7x + 10$.

28. $\frac{a^2 + x - 20}{x - 1}$ by $x^2 + 6x + 5$.

Express:

29. $2\frac{1}{2}$ times $x - 6$.

30. $3\frac{3}{4}$ times $x + 5$.

31. $7\frac{5}{8}$ times $x + 3$.

32. $4\frac{3}{5}$ times $x - 8$.

Multiply:

33. a^2 by $\frac{b}{a}$.

34. $\frac{2x}{y}$ by -1.

35. $3xy$ by $\frac{2}{xy}$.

36. $-\frac{2a}{b}$ by -1.

37. If a boat's crew can row x miles an hour, how far down a river flowing at the rate of 4 miles an hour can they go in one hour? In 2 hours? In $2\frac{1}{3}$ hours?

38. How far could the same crew row up the same river in 1 hour? In 3 hours? In $4\frac{1}{2}$ hours? In 5 hours and 40 minutes?

39. It took this same boat's crew 3 hours to row up the river a distance that they rowed down in one hour; what would be their rate per hour in still water?

40. A boat's crew rowed for 2 hours up a stream flowing at the rate of 2 miles an hour; returning it took them only 1 hour and 12 minutes; how fast can they row?

41. Simplify $\frac{x+2}{x-2} - \frac{x-2}{x+2} + \frac{x-1}{x^2-4}$.

LESSON XXVII.*

Multiplying and Dividing Fractions and Mixed Quantities by Fractions.

Multiply:

1. $\frac{a}{b}$ by $\frac{a}{b}$.
2. $\frac{a}{b}$ by $\frac{b}{a}$.
3. $\frac{3\,a^2x}{5\,by}$ by $\frac{10\,by^2}{21\,a^2x^2}$.
4. $-\frac{7\,x^2y^2}{8\,a^3b}$ by $-\frac{8\,a^3}{7\,x^2y}$.
5. $\frac{x}{y}$ by $\frac{y^2}{x^2}$.
6. $-\frac{4\,x}{5\,a}$ by $-\frac{1}{4}$.
7. $\frac{a^2-b^2}{x^2-4}$ by $\frac{x-2}{a+b}$.
8. $\frac{a^3+b^3}{ab}$ by $\frac{b}{a^2-ab+b^2}$.
9. $\frac{9-a^2}{3+x}$ by $\frac{3+x}{3-a}$.
10. $\frac{a\,(a-b)}{a^2+2\,ab+b^2}$ by $\frac{a^2-b^2}{ab}$.
11. $1+\frac{x}{a}$ by $1-\frac{x}{a}$.
12. $3\,a-\frac{3\,b^2}{a}$ by $\frac{a}{3\,(a+b)}$.
13. $\frac{x^2-9\,x+20}{x^2+7\,x+6}$ by $\frac{x^2+2\,x+1}{x^2-8\,x+16}$.
14. $\frac{a^2+9\,a-10}{a^2-10\,a-11}$ by $\frac{a^2+2\,a+1}{a^2-2\,a+1}$.
15. $a-5-\frac{14}{a}$ by $\frac{a}{a^2-9\,a+14}$.
16. $\frac{x+2}{x+3}$ by $\frac{x+4}{x+5}$.

17. $\frac{x+4}{x-3}$ by $\frac{x-4}{x+3}$.

Divide:

18. $\frac{a}{b}$ by $\frac{a}{b}$.

19. $\frac{x}{y}$ by $\frac{y}{x}$.

20. $\frac{a}{2b}$ by $-\frac{1}{2}$.

21. 1 by $\frac{a}{b}$.

22. -1 by $\frac{x}{y}$.

23. $\frac{4ab}{5xy}$ by $\frac{xy}{ab}$.

24. $2+\frac{4a^2b}{5xy}$ by $\frac{2c}{x^2y^2}$.

25. $3+\frac{a}{x}$ by $3-\frac{a}{x}$.

26. $a-\frac{b^2}{a}$ by $a+2b+\frac{b^2}{a}$.

27. $\frac{a^3+b^3}{a-b}$ by $\frac{a+b}{a^3-b^3}$.

28. $a-b+\frac{b^2}{a+b}$ by $\frac{a}{a^2-b^2}$

29. $\frac{x^2-x-30}{x^2+x-6}$ by $\frac{x^2+8x+15}{x^2-8x+12}$.

30. $\frac{a^2-2ax+x^2}{x+3}$ by $\frac{a^2-2ax+x^2}{x-3}$.

31. $\frac{x-4}{x+1}$ by $\frac{x-2}{x+3}$.

32. $\frac{a-b}{c+d}$ by $\frac{c-d}{a+b}$.

33. $\frac{x-1}{x+7}$ by $\frac{x-8}{x+6}$.

Divide:

34. $a^7 - b^7$ by $a - b$.

35. $x^3 + xy + y^2$ by $x + y$.

36. It took a boat's crew that can row in still water 8 miles an hour, $4\frac{2}{5}$ hours to row back up a stream a distance that they rowed down in two hours; how fast did the stream flow?

LESSON XXVIII.

Equations of the First Degree.

NOTE. The teacher may help on those numbered in full face type. Then the pupils should do the others.

1. $x - (36 - x) = 4$. *Ans.* 20.

2. $x + (7 + x) = -17$. *Ans.* -5.

3. $\frac{2x+1}{2} = \frac{7x+5}{8}$. *Ans.* 1.

4. $\frac{x+2}{2} = \frac{3x+1}{4}$.

5. $\frac{3x+3}{5} + \frac{2x+1}{3} = \frac{5x-2}{3}$.

6. $\frac{5x-11}{4} - \frac{x-1}{10} = \frac{11x-1}{12}$.

7. $x + \frac{19-2x}{3} = \frac{10+x}{2}$.

8. $x + \frac{x+7}{2} - \frac{2x+7}{3} = 4x - 21$.

9. $\frac{x+3}{2} - \frac{x-2}{3} = \frac{x+2}{2} - \frac{1}{6}$.

10. $\frac{4x+3}{10} - \frac{12x-5}{5x-29} = \frac{2x-1}{5}$.

11. $\frac{9x+20}{36} - \frac{4x-12}{5x-4} = \frac{x}{4}$.

12. $\frac{2x-8}{3} - \frac{5x-2}{x+1} = \frac{4x+7}{6}$.

13. $\frac{3x+7}{4} + \frac{2x+1}{3x+3} = \frac{3x+11}{4}$.

14. $\frac{5x+7}{3x+2} = \frac{5x-5}{3x-4}$. *Ans.* 3.

15. $\frac{3x}{4x+1} + \frac{x}{4x-2} = 1$. *Ans.* 2.

16. $\frac{8x+7}{5x+4} = 2 - \frac{2x}{5x+1}$.

17. $\frac{1}{x-7} - \frac{1}{x-4} = \frac{1}{x-1} - \frac{1}{x+2}$. *Ans.* $2\frac{1}{2}$.

NOTE. First simplify the members separately.

18. $\frac{1}{x-1} - \frac{1}{x-3} = \frac{1}{x+2} - \frac{1}{x}$ *Ans.* $\frac{2}{3}$.

19. $\frac{1}{x-7} - \frac{1}{x+7} = \frac{1}{x-9} - \frac{1}{x+5}$. *Ans.* 1.

20. $\frac{1}{x-10} - \frac{1}{x-7} = \frac{1}{x-9} - \frac{1}{x-6}$.

21. $\frac{1}{x+3} - \frac{1}{x+1} = \frac{1}{x+7} - \frac{1}{x+5}$.

22. $\frac{x+5}{x+4} - \frac{x-6}{x-1} = \frac{x-4}{x+5} - \frac{x-15}{x-16}$.

23. $\frac{x+3}{x+6} - \frac{x+6}{x+9} = \frac{x+2}{x+5} - \frac{x+5}{x+8}$.

24. $\frac{x-7}{x-9} - \frac{x-9}{x-11} = \frac{x-13}{x-15} - \frac{x-15}{x-17}$.

25. $\frac{x+2}{x} - \frac{x+3}{x+1} = \frac{x-6}{x-4} - \frac{x-7}{x-5}$.

LESSON XXIX.

Problems of the First Degree Solved by the Use of One Letter.

NOTE. If the teacher has to give any help on these problems, it should be on those numbered in the heavier type as 4, 8, etc., only.

1. Divide a string 42 inches long into two parts such that one of them will be three-fourths the length of the other. *Ans.* 24 and 18 inches.

2. The sum of two numbers is 85, and the difference 7; what are the numbers?

3. The sum of two quantities is a, the difference b; what are the quantities? *Ans.* $\frac{a+b}{2}$ and $\frac{a-b}{2}$.

4. A can do a pieee of work in 10 days and B in 8; how long will it take them both to do it?

5. A and B can do a piece of work in 5 days and B alone in 9 days; how long will it take A alone to do it? *Ans.* $11\frac{1}{4}$ days.

6. It takes A twice as long as it does B to do a piece of work, and both together can do it in 4 days; how long would it take each? *Ans.* A in 12 days and B in 6.

7. A can do a piece of work in a days and B in b days; how long would it take both to do it?

8. A workman was engaged to work for thirty days with the agreement that for every day he worked he was to receive $2.00, and for every day he was absent he would forfeit fifty cents; he received $47.50; how many days did he work?

9. A man expends one-fifth his income for board, three-twentieths for lodging, one-tenth for clothes, three-eighths for miscellaneous expenses, and had left $175; what was his income?

10. A man is three times as old as his son, but in 5 years more he will be only $2\frac{1}{2}$ times as old; how old are they each?

11. Two casks contain an equal quantity of oil, but when 5 gallons were drawn from one and 25 from the other, the first contained three times as much as the other; how many gallons were there in each at first?

12. To a certain number add 18, and also from the same number subtract 2; three times the sum equals nine times the difference; what is the number?

13. The difference between two numbers is 6, and if 27 be added to their sum, it will give 29; what are the numbers?

14. The sum of two numbers is 16, and if 12 be added to their difference, the result will be 20; what are the numbers?

15. After paying the fourth and the fifth of a debt, $38.50 is still due; what was the debt?

16. After paying the fifth of a debt, and one-third of the remainder, $48.00 is still due; what was the debt?

17. After paying the half of a debt, one-third of the remainder, and one-fourth of what was then left, and

lastly \$7.00, there was still due \$5.00; what was the original debt?

18. After selling a third of his sheep, a fourth of the remainder, and a third of what he then had, a farmer had 20 sheep left; how many had he at first?

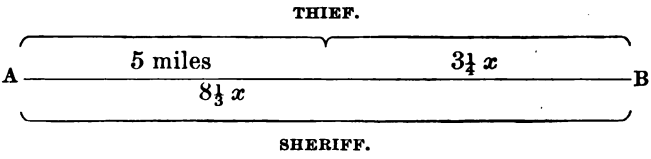
19. The difference between the squares of two consecutive numbers is 25; what are the numbers?

20. The difference between the squares of two consecutive even numbers is 28; what are the numbers?

21. The difference between the squares of two numbers, one of which is 3 larger than the other, is 3; what are the numbers?

22. A thief travelling at the rate of $3\frac{1}{4}$ miles an hour is 5 miles ahead of the sheriff pursuing at the rate of $8\frac{1}{3}$ miles an hour; how long before the former will be overtaken?

Illustrate thus on a scale:



23. An hour after a freight train going at the rate of 18 miles an hour has left the station, it is followed by an express going at the rate of 42 miles an hour; how long before the express will overtake the freight?

24. A steamboat going at the rate of $9\frac{1}{3}$ miles an hour is 7 miles ahead of another going at the rate of $11\frac{2}{3}$ miles per hour; how long before the second will be 7 miles ahead of the first?

25. The minute-hand of a watch goes one minute space per minute, and the hour-hand goes $\frac{1}{12}$ of a minute space per minute; at 4 o'clock, the hour-hand has a lead of 20 spaces; how long before the minute-hand overtakes it?

Ans. $21\frac{9}{11}$ minutes.

26. The hour and minute hands of a watch are together at 12 o'clock; when will they be together next.

Ans. At $5\frac{5}{11}$ minutes past one.

27. The hour and minute hands of a watch are together at 12 o'clock; how long before they will be pointing in opposite directions?

28. At what time between five and six o'clock will the hour and minute hands of a clock be together?

29. At what time between one and two o'clock will the two hands be pointing in opposite directions?

30. A hare is 80 of her own leaps ahead of a greyhound, and takes 3 leaps while the greyhound takes 2; but one of the greyhound's leaps equals two of hers; how many leaps will the hare take before she is caught?

31. A hare is 45 of her leaps ahead of a hound, and is taking five leaps to the hound's 4; but 2 of the hound's leaps equal 3 of the hare's; how many leaps will the hare take before the hound catches her? *Ans.* 225.

32. A hare is 70 of her leaps ahead of a hound, and 280 more leaps will put her in a place of safety. She takes three leaps to the hound's 2, but one of the hound's is equal to 2 of hers; will she be caught?

33. A man is 30 of his own steps behind his son, and takes 3 steps to his son's 5; but one of his steps equals 2 of his son's; how many steps must he take before he overtakes his son?

34. A certain number of persons paid a bill of \$1.20; if there had been one more of them, each would have paid 10 cents less; how many were there?

35. A boy bought a number of apples for 18 cents, but had to throw away three of them, making the others cost him half a cent apiece more; how many did he buy?

LESSON XXX.

Equations Using Two or More Letters.

1. $x + y = 16.$
$x - y = 4.$

2. $ax + by = m.$
$ax - by = n.$

3. $x + y = 6.$
$x - y = 20.$

4. $x + y = 5.$
$3x - 2y = 30.$

5. $6x + y = 32.$
$2x - 7y = -4.$

6. $8x - 3y = 5.$
$7x - 2y = 5.$

7. $\frac{5x + 2y}{13} = 2.$
$\frac{4x - 2y}{5} = 2.$

8. $\frac{3x + 5y}{2} = \frac{7x + 20}{3}.$
$\frac{2x - y}{9} = \frac{5y - 20}{5}.$

9. $\frac{4x + 7y}{4} = 1.$
$\frac{5x - 2y}{23} = 2.$

10. $x + y + z = 9.$
$3x - 2y - z = 8.$
$2x - y + z = 8.$

11. $x - y + 2z = 2.$
$2x - 3y + z = -9.$
$3x + y - 8z = 6.$

12. $x + y = 11.$
$x + z = 13.$
$y + z = 12.$

13. $5x + z = 10.$
$2x + 3y = 8.$
$2y - z = 1.$

14. $2x - 3y + 2z = 9.$
$x - z = -1.$
$6y - 3z = 3.$

15. $(x + 3)(y + 5) = xy + 32.$
$(x + 1)y = xy + 2x - 8.$

16. $\frac{x+y}{x-y-2} = \frac{1}{2}.$
$\frac{x-y}{x+y+4} = \frac{1}{2}.$

17. $\frac{2x-3y-3}{3x-y} = \frac{1}{3}.$
$\frac{x-3y}{x-5} = \frac{1}{3}.$

18. $\frac{1}{x} + \frac{1}{y} = \frac{7}{10}.$
$\frac{1}{x} - \frac{1}{y} = \frac{3}{10}.$

NOTE. Eliminate one of the unknown quantities before clearing of fractions.

19. $\frac{3}{x} + \frac{5}{y} = 2.$
$\frac{6}{x} + \frac{7}{y} = 2.$

20. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{13}{12}.$
$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = \frac{13}{12}.$
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} = 0.$

21. $\frac{1}{x} - \frac{1}{y} = \frac{1}{12}.$
$\frac{1}{y} - \frac{1}{z} = \frac{1}{8}.$
$\frac{1}{y} + \frac{1}{z} = \frac{7}{24}.$

22. $x + y + z = 0.$
$3x - y + 5z = 2.$
$2x - y + 4z = 0.$

23. $\frac{a}{x} + \frac{a}{y} = \frac{7a}{12}.$
$\frac{a}{x} + \frac{a}{z} = \frac{a}{2}.$
$\frac{a}{y} - \frac{a}{z} = \frac{a}{12}.$

LESSON XXXI.

Problems.

NOTE. Use two letters to express the unknown quantities. Have pupils prove their answers where answers are not given.

1. The sum of two numbers is 43, and their difference 11; what are the numbers?

2. A boat's crew rowed up a river 11 miles in 2 hours, and down the same river, $10\frac{1}{2}$ miles in 1 hour; how fast can the crew row, and what is the velocity of the stream? *Ans.* Crew rows 8 miles per hour. Stream flows $2\frac{1}{2}$ miles per hour.

3. A steamboat on a river makes two landings 21 miles apart. Going down, her time between the landings is 1 hour and 30 minutes, but going up it is 2 hours and 6 minutes; what is her speed, and what is the velocity of the current? *Ans.* Steamboat 12 miles. Current 2 miles.

4. A farmer sold 20 bushels of wheat and 25 bushels of barley for \$32.25, and 50 bushels of wheat and 5 bushels of barley for \$43.25; what was the price of each?

5. A farmer sold 3 horses and 4 cows for \$340, and at the same price 2 horses and 1 cow for \$185; what was the price of each?

6. A number consists of two digits whose sum is 11; if 63 be added to the number the figures will be reversed; what is the number? *Ans.* 29.

7. The sum of the two digits of a number is 12, and if 36 be subtracted from the number, the digits will be reversed; what is the number?

8. The difference between the two digits of a number is 3, the 10's figure being the larger, and if 2 more than five times the sum of the digits be subtracted from the number, the digits will be reversed; what is the number?

9. If 3 be added to the denominator of a certain fraction, it becomes $\frac{1}{3}$; if 2 be added to the numerator, it becomes $\frac{2}{3}$; what is the fraction? *Ans.* $\frac{4}{9}$.

10. If 2 be added to the denominator of a certain fraction, it becomes $\frac{1}{3}$; if 1 be subtracted from the denominator, it becomes $\frac{1}{2}$; what is the fraction?

11. A certain fraction becomes $\frac{1}{2}$ by adding 1 to the numerator, and $\frac{1}{3}$ by adding 1 to the denominator; what is the fraction?

12. A and B together can do a piece of work in 6 hours, but A worked 7 hours and B finished it in 4; how long would it take each alone? *Ans.* A 9 hours, and B 2.

13. A and B can do a piece of work in 2 hours and 24 minutes; and they can also do it if A works 3 hours and B 2; how long would it take each alone?

LESSON XXXII.*

Involution Continued — Application of the General Formulas for the Binomial Theorem.

General Formulas:

$$(a \pm b)^2;\ (a \pm b)^3;\ (a \pm b)^4, \text{ etc.}$$

Expand:

1. $(2a + 3)^2$.

2. $(3 - 3b)^2$.

3. $(2x + 3y)^3$.

4. $(3x - 2y)^4$.

5. $(1 - 2x)^5$.
6. $(3x + 2y)^5$.
7. $(5x - 2y)^3$.
8. $(a^2 - b^2)^4$.
9. $(x^2 - y^2)^3$.
10. $(a^2 + b^3)^5$.
11. $(x^2y - 2)^4$.
12. $(x^3 + y^3)^3$.
13. $(xy - a)^3$.
14. $(ax^2 + bx)^3$.
15. $(a^2x + b^2y)^2$.
16. $(2ax - 2ay)^2$.
17. $\left(\frac{x}{2} + y\right)^3$.
18. $\left(\frac{x}{2} + 2y\right)^2$.
19. $\left(\frac{2x}{3} + \frac{3y}{2}\right)^2$.
20. $(a + (b + c))^2$.
21. $(m + n + r)^2$.
22. $(2x + 3y + z)^2$.
23. $(3a + b + 2)^2$.
24. $(x + 2y + 3z)^2$.
25. $(a + b - c)^2$.
26. $(a - b + c)^2$.
27. $(a - b - c)^2$.
28. $(43)^2 = (40 + 3)^2$.
29. $(56)^2$.
30. $(37)^2$.

Expand (mentally):

31. $(49)^2 = (50 - 1)^2$.
32. $(59)^2$.
33. $(68)^2$.
34. $(37)^2$.
35. $(51)^2$.

LESSON XXXIII.*

Evolution, Square Root — Pure Equations of the Second Degree.

Formula:

$$(a + b)^2 = a^2 + 2ab + b^2 = a^2 + (2a + b)b.$$

Extract the square root of

1. $x^2 + 6x + 9$.
2. $x^2 - 8x + 16$.

3. $4x^2 + 12xy + 9y^2$.
4. $25x^4 - 20x^2y + 4y^2$.
5. $49a^4 - 42a^2 + 9$.
6. $81x^4 + 126a^2b^2 + 49b^4$.
7. $64x^6 - 160x^3y^2 + 100y^4$.
8. $36a^2x^4 + 36abx^2y + 9b^2y^2$.
9. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

Find roots of the following:

10. $\sqrt{1225}$.
11. $\sqrt{5625}$.
12. $\sqrt{8649}$.
13. $\sqrt{54756}$.
14. $\sqrt{15625}$.
15. $\sqrt{2}$ (to two decimal places).
16. $\sqrt{3}$ (to two decimal places).
17. $\sqrt{5}$ (to two decimal places).
18. $\sqrt{7}$ (to two decimal places).
19. $\sqrt{8}$ (to two decimal places).

Find the value of x and y:

20. $x^2 = 144$. *Ans.* ± 12.
21. $x^2 = 81$.
22. $x^2 + 7 = 56$.
23. $x^2 - 3a^2 = 22a^2$. *Ans.* $\pm 5a$.
24. $x^2 + y^2 = 117$.
    $x^2 - y^2 = 45$.
25. $5x^2 - y^2 = 109$.
    $x^2 + y^2 = 41$.
    Ans. $x = \pm 5$. $y = \pm 4$.
26. $4x^2 + 2xy + 7 = x^2 + 2xy + 307$.
27. $x^2 = 15$. *Ans.* $x = 3.87 +$.
28. $x^2 = 18225$.
29. $x^2 - 37636 = 0$.
30. $\sqrt{x} = 5$.
31. $\sqrt[3]{x} = 7$.
32. $\sqrt{2x} = 16$.
33. $\sqrt[3]{3ax} = 6a$.
34. $\sqrt[4]{x} = 2a$.
35. $\sqrt{8x} = 4$.
36. $\sqrt[3]{3x} = 6$.
37. $\sqrt{5x} = 15$.

38. $\sqrt[3]{\frac{x}{2}} = 2.$ **39.** $\sqrt{\frac{x}{3}} = 12.$

40. Give the six axioms chiefly used in solving equations.

LESSON XXXIV.*

Evolutions, Cube Root — Equations of the Third Degree.

Formula:

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^2 + (3a^2 + 3ab + b^2)\,b.$

Cube root of:

1. $x^3 + 3x^2y + 3xy^2 + y^3.$

2. $a^3 - 18a^2 + 108a - 216.$

3. $8a^3 + 36a^2 + 54a + 27.$

4. $8x^3 - 36bx^2 + 54b^2x - 27b^3.$

5. $125x^3 + 150x^2y + 60xy^2 + 8y^3.$

6. $343 - 294a + 84a^2 - 8a^3.$

Extract:

7. $\sqrt[3]{74088}.$ **10.** $\sqrt[3]{21024576}.$

8. $\sqrt[3]{132651}.$ **11.** $\sqrt[3]{5}$ (to one decimal place).

9. $\sqrt[3]{57912}.$ **12.** $\sqrt[3]{12}$ (to one decimal place).

Find the value of x and y:

13. $x^3 = 343.$ **15.** $x^3 - 13824 = 0.$

14. $x^3 = -729.$ **16.** $x^3 = 512a^6.$

17. $x^3 + y^3 = 35.$ *Ans.* $x = 3.$

$x^3 - y^3 = 19.$ $y = 2.$

18. $x^3 + y^2 = 368.$
$x^3 - y^2 = 318.$

19. $x^3 + xy = 270.$
$x^3 - xy = 162.$

Expand:

20. $(2a - b)^5.$

21. $\left(\frac{a}{2} + 4\right)^4.$

22. $\left(\frac{x}{3} + 3y\right)^3.$

23. $\left(2b + \frac{a}{2}\right)^4.$

24. $(2x^2 - 3y^2)^3.$

25. $(x^2 + y^2)^5.$

LESSON XXXV.*

Roots of Equations of the First and Second Degrees.

What value of x will satisfy the following equations?

1. $x - 5 = 0.$

2. $x + 4 = 0.$

3. $x - 7 = 0.$

4. $x + 1 = 0.$

5. $x - 1 = 0.$

6. $3x - 4 = 0.$

7. $2x - 4a = 0.$

8. $5x + 15 = 0.$

What values of x will satisfy the following?

9. $(x + 1)(x - 2) = 0.$

10. $(x + 3)(x + 5) = 0.$

11. $(x - 8)(3x - 6) = 0.$

12. $(5x + 15)(3x - 9) = 0.$

13. $(x - 4)(x - 3) = 0.$

14. Of what degree are equations 1 to 8? How many roots has each?

15. Of what degree are equations 9 to 13? (Determine by multiplication.) How many roots has each?

What values of x will satisfy the following?

16. $(x + 1)(x - 2)(x - 3) = 0.$

17. $(x - 2)(x + 2)(x - 4) = 0.$

18. Problems 16 and 17 are equations of what degree? How many roots has each?

Factor the first member of the following equations, and determine the roots, or values of x; but first, when necessary, make the second member 0 by eliminating from it every positive or negative quantity: —

19. $x^2 - 7x + 12 = 0.$ *Ans.* 3, or 4.

20. $x^2 - 2x - 48 = 0.$ *Ans.* 8, or -6.

21. $x^2 + x - 42 = 0.$

22. $x^2 - 5x = 6.$

23. $x^2 + 2x = -1.$

24. $x^2 - 4x + 4 = 0.$

25. $x^2 - 18x + 80 = 0.$

26. $x^2 - 6ax + 8a^2 = 0.$

Factoring; Case XI.

General Formulas:

1. $a^2x^2 + (b + c)ax + bc = (ax + b)(ax + c).$
2. $a^2x^2 - (b + c)ax + bc = (ax - b)(ax - c).$
3. $a^2x^2 + (b - c)ax - bc = (ax + b)(ax - c).$

Factor:

27. $9x^2 - 24x + 15.$

28. $25x^2 + 15x + 2.$

29. $16x^2 - 12x - 10.$

30. $4x^2 + 6x - 18.$

Factor the first member of the following equations, and determine the roots, or values of x: —

31. $9x^2 - 24x + 15 = 0$. *Ans.* 1, or $1\frac{2}{3}$.

32. $36x^2 - 30x - 6 = 0$. *Ans.* 1, or $-\frac{1}{6}$.

33. $4x^2 + 40x + 96 = 0$.

34. $25x^2 + 25x - 50 = 0$.

35. $16x^2 - 16x - 32 = 0$.

36. $49x^2 - 35x - 14 = 0$.

37. $b^2x^2 - 3b^2x + 2b^2 = 0$.

For Case XII., see Lesson XXXIX.

Extract:

38. $\sqrt{36x^2 - 60xy + 25y^2}$.

39. $\sqrt[3]{8x^3 + 36x^2 + 54x + 27}$. *Ans.* $2x + 3$.

40. $\sqrt{x^4 + 4x^3 - 2x^2 - 12x + 9}$.

Ans. $x^2 + 2x - 3$.

LESSON XXXVI.

Affected Quadratic Equations Solved by Completing the Square.

Find the

1. $\sqrt{x^2 + 8x + 16}$.

2. $\sqrt{x^2 - 7x + 12\frac{1}{4}}$.

3. $\sqrt{x^2 + 3x + 2.25}$.

Add a third term to the following binomials that will make them perfect squares, and extract the root: —

4. $x^2 + 10x$.

5. $x^2 + 12x$.

6. $x^2 - 6x$.

7. $x^2 + 5x$.

8. $x^2 - 9x$.

9. $x^2 + 4ax$.

In the same way, add to both members of each of the following equations a quantity that will make the first member a perfect square, and then find the roots of the equation: —

10. $x^2 + 4x = 45$.

11. $x^2 - 14x = -48$.

12. $x^2 + 5x = 24$.

13. $4x^2 - 12x = 40$.

14. $9x^2 + 18x = 72$.

Find the roots of the following equations to two decimal places: —

15. $x^2 + 6x = 42$.

16. $x^2 - 7x = 20$.

Find the roots of the following equations, either by completing the square, or by factoring as in the preceding lesson: —

17. $x^2 + 14x = -49$. *Ans.* -7 and -7.

18. $x^2 - 11x = 30$. *Ans.* 5, and 6.

19. $9x^2 - 15x - 14 = 0$.

20. $x^2 - 8x + 15 = 0$.

21. $3x^2 + 6x - 24 = 0$.

22. $2x^2 - 4x = 30$.

23. $25x^2 - 20x - 60 = 0$.

24. $x + 3 = \frac{40}{x}$.

25. $x - 5 = \frac{6}{x}$.

26. $x + 2 = \frac{20}{x+1}$.

27. $x = \frac{63}{x+2}$

28. $2x + 5 = \frac{273}{2x-3}$.

29. $3x - 10 = \frac{28}{3x+2}$.

30. $\frac{5}{x} + x = 6.$

31. $\frac{x+2}{x-2} = \frac{2x+7}{x+1}.$

32. $\frac{x+4}{3x-8} = \frac{x+5}{2x-1}.$

LESSON XXXVII.

Problems Producing Affected Quadratic Equations.

NOTE. In a few cases it may be interesting to try to interpret the negative value of x, but as a rule it should be disregarded.

1. A rectangular board is 5 inches longer than it is wide, and contains 234 square inches; what are its dimensions? *Ans.* 13 by 18.

2. Divide the number 17 into two such parts that their product will be 52.

3. There are two numbers whose difference is 9, and their sum multiplied by the greater is 266; what are the numbers? *Ans.* 14 and 5.

4. A field 4 rods longer than wide contains 140 sq. rods; what are the dimensions of the field?

5. Divide the number 19 into two such parts that their product will be 84.

6. A boy bought some oranges for 24 cents; if he had got 2 more for the 24 cents, the oranges would have cost one cent apiece less; how many did he buy?

7. A boy bought some apples for 12 cents; losing 2 of them, it made the others cost him a cent apiece more; how many did he buy? *Ans.* 6.

8. A woman bought some eggs for 40 cents, but losing 4 of them, the rest cost her one-half a cent apiece more; how many did she buy?

9. The sum of the squares of two consecutive numbers is 313; what are the numbers?

NOTE. Compare with Problem 19, Lesson XXIX.

10. The sum of two consecutive even numbers is 580; what are the numbers?

11. Find a number such that if 15 be added to it its square the sum will equal 8 times the number.

12. By going 2 miles an hour more, A went 80 miles in 2 hours less than B; at what rate did each travel?

LESSON XXXVIII.

Arithmetical Problems solved by Algebraic Formulas.

NOTE. Already there has been some work of this kind, as in extracting square root, Lesson XXXIII., and cube root, Lesson XXXIV. The following problems will give a little more extended practice of this kind. First work out the general formula, and then do the arithmetical problems by it.

I. A can do a piece of work in a days (or hours), and B can do it in b days (or hours); how long will it take them to do it together?

Solution:

Let x be the time it will take them both; then,

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{x}.$$

Reducing this gives, $x=\dfrac{ab}{a+b}$.

1. A can do a piece of work in 10 hours, and B in 8; how long will it take both?

2. A in 7 days and B in 5 days (?)
3. A in 4 days and B in 3 days (?)

II. Percentage. *General Formulas.*

Let c be the percentage, b the base, and r the rate; then, when b and r are known.

1. $c = br$.

To find the equation for r when b and c are known; first transpose[1] the members of the above equation, which gives $br = c$; hence,

2. $r = \frac{c}{b}$.

In the same way, find the formula: —

3. $b = \frac{c}{r}$.

Remember the rate numerically must be expressed in hundredths, as 5 per cent is .05, 18 per cent, .18, etc.

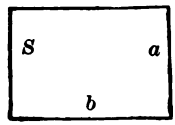
Problems.

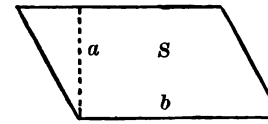
1. 9 per cent of 850.
2. 15 per cent of 620.
3. 13 per cent of 2600.
4. What per cent of 950 is 190?
5. What per cent of 640 is 00?
6. What per cent of 8 is 5?
7. What per cent of 9 is 3?
8. $2\frac{1}{2}$ per cent of what number will give 78?
9. $2\frac{1}{2}$ per cent of what number will give 150?
10. $2\frac{1}{2}$ per cent of what number will give 60?
11. $2\frac{1}{2}$ per cent of what number will give 240?

[1] Transpose is here used in its correct sense; for example, if $6 = x$ then by transposing both members, $x = 6$; but there is no change of signs.

III. Mensuration.

I. *Area of Parallelograms.*





Let S be the surface or area, b the base, and a the altitude.

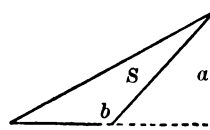
General Formulas.

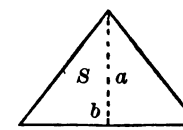
1. $S = qb$.
2. Find it for a, when b and S are given.
3. Find it for b, when a and S are given.

Problems. (Measurements may be called feet, inches, or any unit.)

1. Base 18, altitude 9; area ?
2. Base 25, altitude 6; area ?
3. Base ? altitude 9, area 810.
4. Base 230; altitude ? area 690.
5. Base ? altitude 27, area 100.

II. *Area of Triangles.*





LETTERS AS ABOVE.

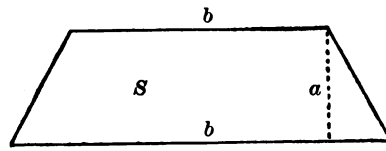
General Formulas.

1. $S = \frac{ab}{2}$.
2. Find it for a, when b and S are given.
3. Find it for b, when a and S are given.

Problems.

1. Base 30, altitude 15; area ?
2. Base 32, altitude 20; area ?
3. Base ? altitude 75, area 900.
4. Base 60; altitude ? area 540.
5. Base 84; altitude ? area 1000.

III. *Area of Trapezoids.*



LETTERS AS BEFORE.

General Formulas.

1. $S = \frac{a\,(b + b')}{2}$.
2. Find it for a when b, b', and S are given.

Problems.

1. Bases 20 and 30, altitude 12; area ?
2. Bases 15 and 35; altitude ? area 500.
3. Bases 9 and 21, altitude 30; area ?

IV. *The Circumference of a Circle.*

Let c be the circumference, R the radius, $2R$ the diameter, and $\pi = 3.1416$, the ratio of the circumference.

General Formulas.

1. $c = 2\pi R$. 2. Find it for $2R$ when c is given.

Problems.

1. Diameter 5; circumference ?
2. Diameter 10; circumference ?

3. Diameter ? circumference 21.9912.
4. Diameter 200; circumference ?
5. Diameter ? circumference 200.
6. Diameter ? circumference 500.

V. *Area of Circle.*

The area of a circle is equal to that of a triangle whose base equals the circumference, and altitude the radius; hence the general formula:—

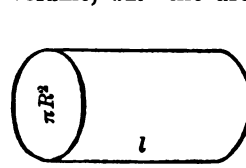
1. $S = 2\pi R \times \frac{R}{2}, = \pi R^2$.
2. Find it for R when s is given.

Problems.

1. Radius 8; area ?
2. Radius 10; area ?
3. Radius ? area 28.2744.
4. Radius ? area 78.54.

VI. *Volume of a Cylinder.*

Let v be the volume, πR^2 the area of one end, and l the length.



General Formulas.

1. $v = l\pi R^2$.
2. Find it for l when v and R are given.

Problems. (Dimensions in inches or feet.)

1. Radius 8, length 15; volume ?
2. Radius 10, length 24; volume ?

3. Radius 6; length ? volume 1500.

4. Radius 3, length 12; volume ?

VII. *Surface of Cylinder.*

General Formula.

1. $S = 2\pi R^2 + 2\pi Rl = 2\pi R(R + l)$.

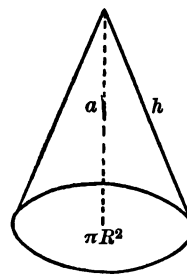
Problems.

1. Radius 5, length 20; surface ?

2. Radius 3, length 9; surface ?

3. Radius 1, length 10; surface ?

VIII. *Volume of a Cone.*



Let πR be the area of base and a the altitude, v the volume.

General Formula. 1. $v = \frac{a\pi R^2}{3}$.

Problems.

1. Radius of base 4, altitude 9; volume ?

2. Radius of base 5, altitude 24; volume ?

IX. *Surface of a Cone.*

Let h be the slant height.

General Formula. (Curved surface.)

1. $S = h\pi R$.

Total Surface S'.

2. $S' = h\pi R + \pi R^2 = \pi R\,(h + R)$.

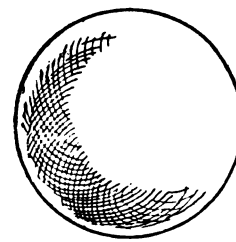
Problems.

1. Radius 3, slant height 8; curved surface?
2. Radius 5, slant height 10; total surface?

X. *Surface of a Sphere.*

General Formula.

1. $S = 4\,\pi R^2$.



Problems.

1. Radius 6; surface?
2. Radius 8; surface?
3. Radius 5; surface?
4. Radius 10; surface?

XI. *Volume of a Sphere.*

General Formula.

1. $v = 4\,\pi R^2 \times \frac{R}{3} = \frac{4\,\pi R^3}{3}$.

Problems.

1. Radius 2; volume?
2. Radius 5; volume?
3. Radius 10; volume?
4. Find the formula for R when the volume is given.

Problems.

1. Volume 37.6992; radius?
2. Volume 50.2656; radius?

LESSON XXXIX.

Factoring. — Case XII. — Permanences and Changes in Signs.

General Formulas:

$$(ax + b)(x + c) = ax^2 + (ac + b)x + bc.$$
$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd.$$

Combine by inspection:

1. $(2x + 3)(x + 2)$.
2. $(3x + 5)(x + 1)$.
3. $(2x - 5)(x - 1)$.
4. $(5x - 6)(x - 7)$.
5. $(4x + 7)(x - 2)$.
6. $(3x - 5)(x + 2)$.
7. $(5x + 8)(x + 1)$.
8. $(7x - 2)(x + 3)$.

Factor:

9. $3x^2 + 5x + 2$.
10. $2x^2 + 3x + 1$.
11. $2x^2 + 11x + 12$.
12. $3x^2 + 16x + 5$.
13. $3x^2 - 9x + 6$.
14. $4x^2 - 9x - 9$.
15. $5x^2 - 33x - 14$.
16. $4x^2 - 3x - 27$.
17. $6x^2 + 13x + 7$.
18. $7x^2 + 5x - 18$.

Find the roots of x in the following equations:

19. $5x^2 - 2x - 16 = 0$.
20. $3x^2 - x - 14 = 0$.
21. $4x^2 + 11x = 3$.
22. $7x^2 + 3x = 22$.
23. $2x^2 + x = 10$.
24. $3x^2 - x - 14 = 0$.
25. $5x^2 + 2x - 16 = 0$.
26. $7x^2 - 3x - 4 = 0$.

Multiply by inspection:

27. $(2x + 3)(3x + 2)$.
28. $(2x + 5)(4x + 3)$.
29. $(3x - 7)(5x - 8)$.
30. $(5x - 9)(2x - 1)$.
31. $(3x - 1)(2x - 1)$.
32. $(2x - 1)(5x + 2)$.

33. $(3x-2)(5x+2)$. **35.** $(3x-3)(5x+4)$.

34. $(5x+2)(3x-3)$. **36.** $(5x-1)(3x-1)$.

37. $(3x-2)(7x-2)$.

Factor:

38. $6x^2+x-2$. **43.** $14x^2+3x-27$.

39. $10x^2-7x-12$. **44.** $15x^2-8x-12$.

40. $8x^2+10x-3$. **45.** $10x^2-19x+7$.

41. $6x^2+x-35$. **46.** $6x^2-x-12$.

42. $8x^2+2x-15$. **47.** $15x^2-16x+4$.

48. $6x^2-19x+15$.

49. The length and breadth of a rectangular field are to each other in the ratio of 5 to 3. If the field was 7 rods longer and 2 rods wider, it would contain 242 square rods; find the length and breadth of the field.

NOTE. Let $5x$ and $3x$ equal the length and breadth respectively.

50. The sides of a rectangular field are in the ratio of 3 to 2. If the length was 5 rods less, and the width 3 rods more, the area would be 361 square rods; find the dimensions of the field.

51. The length of a mat is to its width in the ratio of 4 to 3. If it were 3 feet shorter and 2 feet narrower, it would contain seven square yards; find the dimensions of the mat.

52. $\dfrac{60}{7x-5}+\dfrac{35}{2x-3}=7$.

53. A man bought a number of pounds of coffee. The number of pounds was to the number of cents he paid per pound as 5 to 7. If he had bought 5 pounds more at a price 5 cents per pound less, his coffee would have cost him \$9.00; how much did he buy, and what price did he pay?

54. $\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{5}{6}$. **55.** $\frac{x+4}{x-4} - \frac{x-4}{x+4} = \frac{10}{3}$.

56. $\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$.

57. $\frac{3x-2}{2x-5} - \frac{2x-5}{3x-2} = \frac{8}{3}$.

58. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{13}{6}$. **59.** $5x^2 - 55 = -14x$.

60. $8x^2 + x = 30$. **61.** $4x^2 + 17x = 15$.

62. A train goes 300 miles at a uniform rate; if the rate had been 5 miles an hour more, the time would have been 2 hours less; what is the rate of the train?

63. A man travels 108 miles, and finds that he could have made the journey in $4\frac{1}{2}$ hours less if he had travelled 2 miles an hour faster; at what rate did he travel?

64. Two rectangles contain the same area, 480 square yards. One is 10 yards the longer, and the other is 4 yards the wider; find their sides.

65. A cistern can be filled by two pipes running together in $22\frac{1}{2}$ minutes; the larger pipe can fill the cistern in 24 minutes less than the smaller one; find the time in which each would fill the cistern.

MISCELLANEOUS EXAMPLES.

Multiply:

1. $a^2 - 3a + 2$ by $a - 3$.
2. $9a^2 + 6ab + 4b$ by $3a - 2b$.
3. $x^2 - 3x + 2$ by $x^2 + 3x - 2$.
4. $a + b + c$ by $a - b - c$.
5. $x^2 - y^2 - 2yz - z^2$ by $x^2 - y^2 + 2yz - z^2$.
6. $\frac{a+b}{a} - \frac{a+b}{b}$ by $\frac{a+b}{a} + \frac{a-b}{b}$.
7. $\frac{x-3}{x-2} - \frac{x-5}{x+2}$ by $\frac{x-1}{x+2} - \frac{x-7}{x-2}$.

Divide:

8. $3a^4 - a^3 + 2a^2 + a + 1$ by $a^2 - a + 1$.
9. $a^3 + b^3 + c^3 - 3abc$ by $a + b + c$.
10. $27a^3 - 8b^3$ by $9a^2 + 6ab + 4b^2$.
11. $32x^5 + y^5$ by $16x^4 - 8x^3y + 4x^2y^2 - 2xy^3 + y^4$.
12. $a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 + b^5$ by $a + b$.

Multiply together:

13. $\frac{ab}{xy}$, $\frac{yz}{bc}$, and $\frac{cd}{zv}$.

14. $\frac{a^2bc}{x^2y^2}$, $\frac{ac^3}{xy^3}$, and $\frac{x^4y^5}{a^4bc^3}$.

15. Two gentlemen saw a sum of money lying on a table. One said, "I have five times as much money as there is on that table." The other replied, "I have ten

times as much." Together they had \$105; how much money was there on the table?

16. A company of 90 persons consists of men, women, and children. There are 4 more men than women, and ten more children than adults; how many are there of each?

17. A clerk was five years in the same house. In the first three years he spent \$400 a year, but in each of the following years he spent \$100 more than in the preceding year. At the end of the five years he had saved \$1,700; what was his salary?

18. $\frac{x}{2} + \frac{3x}{5} - \frac{5x}{6} + \frac{7x}{12} = 51$, to find x.

19. $\frac{3x}{4} - \frac{5x}{6} + \frac{x}{12} + \frac{2x}{3} = 64$, to find x.

20. $\frac{x}{a} + \frac{x}{b} + \frac{x}{c} + \frac{x}{d} = m$, to find x.

21. $x + \frac{x}{4} + \frac{x}{5} - \frac{x}{6} = 2x - 43$, to find x.

22. $\frac{ax - b}{4} + \frac{a}{3} = \frac{bx}{2} - \frac{bx - a}{3}$, to find x.

23. A fish was caught whose tail weighed 9 lbs.; its head weighed as much as its tail and half its body, and its body weighed as much as its head and tail together; what was the weight of the fish?

24. A man engaged a workman for 50 days; for each day he worked he received \$1.50, and for each day he was idle he paid 50 cents for board. At the end of the 50 days he received \$45; how many days did he work?

25. An estate of \$7,500 is to be divided between a widow, two sons, and three daughters, so that each son

MISCELLANEOUS EXAM

Multiply:

1. $a^2 - 3a + 2$ by $a - 3$.
2. $9a^2 + 6ab + 4b$ by $3a - 2b$.
3. $x^2 - 3x + 2$ by $x^2 + 3x - 2$.
4. $a + b + c$ by $a - b - c$.
5. $x^2 - y^2 - 2yz - z^2$ by $x^2 - y^2$.
6. $\frac{a+b}{a} - \frac{a+b}{b}$ by $\frac{a+b}{a} + \frac{a}{}$
7. $\frac{x-3}{x-2} - \frac{x-5}{x+2}$ by $\frac{x-1}{x+2} -$

Divide:

8. $3a^4 - a^3 + 2a^2 + a + 1$ l
9. $a^3 + b^3 + c^3 - 3abc$ by a
10. $27a^3 - 8b^3$ by $9a^2 + 6a$
11. $32x^5 + y^5$ by $16x^4 - 8$
12. $a^5 + a^4b + a^3b^2 + a^2b^3 +$

Multiply together:

13. $\frac{ab}{xy}$, $\frac{yz}{bc}$, and $\frac{cd}{zv}$.

15. Two gentlemen
table. One
there is o

$x^3 - 12x + 5$.
$x^3 - 40x^2 + 9$.
$x^3 + 5x^2 - 20$.
$- 8y^3$.

-3

-8, and $\frac{x^2+5x}{x+1}$.

$+12x-64$, and divide the

x^3-64

$16x+64$

$+4x+16$.

by $\frac{x^2-6x-7}{2x^2-17x+21}$, and divide

t by $\frac{4x^2-20x+25}{4x^2-1}$.

$^3-x^2-41x+105$, and x^4-2x^3-

the greater by the less, and factor the remainder

D. of $x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4$

$-x^2y-xy^2+y^3$.

G. D. of $x^4-x^3+8x-24$, and x^3+4x^2-8x

57. G. C. D. of $x^3 + 3x^2 - 8x - 24$, and $x^3 + 3x^2 - 3x - 9$.

58. G. C. D. of $x^4 - 3x^3 + 2x^2 + x - 1$, and $x^3 - x^2 - 2x + 2$.

59. Simplify $\frac{a^2 - b^2}{a^2 - 3ab + 2b^2} \times \frac{ab - 2b^2}{a^2 + ab} \times \frac{a^2 - ab}{a^2 - 2ab + b^2}$.

60. $x^2 - 15x + 54$; find roots of the equation.

61. $4x^2 + 12x = 40$; to find the roots.

62. $(x + 5)^2 + 8(x + 5) = 180$; find the roots.

63. $x^2 - 4ax = 5a^2$; find the roots.

64. A number consists of two digits whose sum is 13; if 45 be added to the number, the figures will be reversed; what is the number?

65. A man paid out one-fourth of the money he had, and afterwards received \$3. Again he paid out a third of what he then had, and afterwards received \$2. Lastly he paid out one-eighth of what he then had, and found he had \$14 left; how much had he at first?

66. A farmer, wishing to sell his flock of sheep, sells half of the whole number and half a sheep over. Next he sells half of what he had left and half a sheep over. This he does a third and fourth time, disposing at each sale of half the sheep left on his hands and half a sheep over. A fifth sale of eight sheep disposed of the entire flock; how many sheep had he at first?

67. If two apples cost one cent and three pears two cents, and I buy 100 apples and pears for 56 cents, how many of each do I buy?

68. $5x + \frac{y}{2} = 24.$
$\frac{3x}{2} - \frac{y}{4} = 4.$

69. $\frac{x+3}{4} - y = -2.$
$\frac{x-3}{4} + 4y = 21\frac{1}{2}.$

70. A owes \$1200, and B owes \$2500, but neither has enough to pay his debts. Said A to B, "Lend me the eighth of what you have and I can pay my debt."—"Lend me," replied B, "the ninth part of what you have, and I can pay mine." How much money had each?

71. A man being asked the ages of himself and his son, replied: "Six years ago I was three times as old as he, but three years hence I shall be only $2\frac{1}{8}$ times as old as he." What is the age of each?

72. At what time between 10 o'clock and 11 o'clock will the minute and hour hand of a clock be together.

73. 34 lbs. of zinc loses 5 lbs. when immersed in water, and 17 lbs. of tin loses 2 lbs. in water. A composition of zinc and tin weighing 136 lbs. loses 19 lbs. in water. How much of each does the composition contain?

74. A certain fraction becomes $\frac{1}{2}$ when 4 is added to the numerator, and $\frac{1}{4}$ when 2 is subtracted from the denominator; what is the fraction?

75. A and B together buy 150 acres of land; if A had bought twice as much and B only three-fourths as much, they would have together bought 50 acres; how many acres did each buy?

Interpret negative result.

76. Find the square root of $4x^2 + 20xy + 12x + 25y^2 + 30y + 9$.

77. The cube root of $8\,a^6b^3 - 36\,a^5b^4 + 54\,a^4b^5 - 27\,a^3b^6$.

78. The square root of $9\,x^4 - 24\,x^3 + 46\,x^2 - 40\,x + 25$.

79. The square root of $a^4x^6 + 2\,a^3x^5 + a^2x^4 - 4\,a^3x^4 - 4\,a^2x^3 + 4\,a^2x^2$.

80. The cube root of $a^3x^6 - 3\,a^5x^5 + 3\,a^7x^4 - a^9x^3$.

81. The square root of $4\,x^6 - 12\,x^5 + 13\,x^4 - 6\,x^3 + x^2$.

82. The square root of $25\,x^4 - 30\,x^3y - 11\,x^2y^2 + 12\,xy^3 + 4y^4$.

83. The cube root of $8\,a^3x^3 - 36\,a^2bx^2y + 54\,ab^2xy^2 - 27\,b^3y^3$.

Simplify the following:

84. $\dfrac{5}{x^2 - 5x + 6} + \dfrac{5}{x^2 + x - 6}$.

85. $\dfrac{x+2}{x-5} - \dfrac{x-3}{x+5} + \dfrac{x+8}{x+2}$.

86. $\dfrac{2\,x^2}{x^2 - y^2} - \dfrac{2\,x}{x+y} + \dfrac{4\,y}{x-y} - \dfrac{y}{x+y}$.

87. $\dfrac{1}{2\,a - 8\,x} - \dfrac{a}{3\,a^2 - 48\,x^2} + \dfrac{1}{2\,a + 8\,x}$.

88. $\dfrac{3}{8\,(a-x)} + \dfrac{1}{4\,(a+x)} - \dfrac{a-x}{4\,(a^2 - x^2)}$.

89. $\dfrac{24\,x}{9 - 12\,x + 4\,x^2} - \dfrac{3 + 2\,x}{3 - 2\,x} + \dfrac{3 - 2\,x}{3 + 2\,x}$.

Find the roots of the following equations:

90. $5\,x^2 - 75\,x + 250 = 0$.

91. $3\,x^2 + 2\,x - 8 = 0$.

92. $2x^2 - 7ax + 3a^2 = 0.$

93. $4x^2 - 4x - 48 = 0.$

94. $9x^2 + 6x - 63 = 0.$

95. $a^2x^2 - 2ax - 120 = 0.$

96. $4x^2 - 4ax - 15a^2 = 0.$

97. $\dfrac{2x-4}{x+1} = \dfrac{x+5}{x+13}.$

98. $\dfrac{5x-7}{x+1} = \dfrac{2x+6}{x+3} + 1.$

99. $\dfrac{5x+9}{x+5} - \dfrac{3x+3}{x+3} = 1.$

100. $\dfrac{7x+4}{x+1} - \dfrac{x+2}{x-1} = 2.$

101. $\dfrac{3x-4}{2x-4} = \dfrac{6x}{x+8}.$

102. $\dfrac{3x-1}{4x+7} = 1 - \dfrac{6}{x+7}.$

103. $\dfrac{x+3}{2x-7} - \dfrac{2x-1}{x-3} = 0.$

104. $x + y = 28.$
$xy = 187.$

105. $x^2 + y^2 = 58.$
$xy = 21.$

106. $x^2 - y^2 = -40.$
$x - y = -2.$

107. $9x^2 - 4y^2 = 864.$
$3x + 2y = 36.$

108. $8x^2 + 18y^2 = 272.$
$xy = 10.$

109. A man sold a number of sheep for \$168. If he had sold 14 more for the same sum he would have received one dollar less for each sheep; how many sheep did he sell?

Interpret the negative answer.

110. Find a number such that four times its square plus nine times the number itself will equal $7\frac{1}{2}$.

Interpret the negative result.

111. A rectangular field is 5 rods longer than it is wide, and its area is 204 sq. rods; what is the length and width of the field?

112. A man divided \$2.40 among a number of boys. If there had been three boys more each would have received 18 cents less; how many boys were there?